New results on the chiral crossover in the twisted-mass approach with 2+1+1 flavors and new physics perspectives

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Meeting of the Working Group on
Theory of Hadronic Matter under Extreme Conditions

Dubna, October 31 – November 3, 2016

The tmfT Collaboration
(“twisted mass at finite temperature”) :

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M. P. Lombardo (INFN, Lab. Naz. Frascati)

M. Müller-Preussker (HU Berlin)

M. Petschlies (The Cyprus Institute)

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Presently active :

F. Burger, E.-M. Ilgenfritz, M. P. Lombardo, A. Sternbeck, A Trunin

Outline:

1. **Brief History of Lattice QCD in Dubna**
2. **Motivating the Running Project : Twisted Mass Approach at** $T \neq 0$
3. **Simulation Setup to Find the Crossover Temperatures for** $N_f = 2$
4. **Towards the Chiral Limit of** T_χ **for** $N_f = 2$
5. **Simulation Setup for** $N_f = 2 + 1 + 1$ **flavors**
6. **The Equation of State for** $N_f = 2$ **and** $N_f = 2 + 1 + 1$
7. **Lattice and Continuum QCD : Landau gauge Propagators at** $T \neq 0$
8. **Perspectives of the Project: new Questions/new Partners for Dubna**
9. **Conclusions and Outlook**

1. Brief History of Lattice QCD in Dubna

Lattice simulations in Russia have begun 1981/82 simultaneously in JINR (BLTP and LIT) and ITEP (led by $\boxed{\text{M. I. Polikarpov}}$).

Supported by D. V. Shirkov (BLTP) and M. G. Meshcheryakov (LIT).

The JINR task force group of the first years :

- V. P. Gerdt (JINR LIT)
- E.-M. Ilgenfritz (JINR BLTP, from Leipzig, then GDR)
- V. K. Mitrjushkin (JINR BLTP)
- M. Müller-Preussker (JINR BLTP, from Berlin, then GDR)
- A. M. Zadorozhny (JINR LIT)

Collaborators that have joined us (mostly temporarily):

- **A. S. Ilchev (JINR BLTP, from Sofia, Bulgaria)**
- **N. V. Makhaldiani (JINR LIT)**
- **S. Yu. Shmakov (JINR LIT)**
- **I. K. Sobolev (JINR BLTP)**
- **I. L. Bogolubsky (JINR LIT, still active "on the lattice")**
- **V. G. Bornyakov (IHEP Protvino, now also at ITEP Moscow and FEFU Vladivostok)**

Collaboration with the home institutes of the German members :

- **Universität Leipzig (J. Ranft, A. Schiller, J. Kripfganz)**
- **Humboldt Universität Berlin (MMP has later built his own group)**

A political-satirical performance given in Dubna by MMP and EMI. Annual party of the East German community in JINR, 1981

Collaboration of the German-Russian group with other institutes :

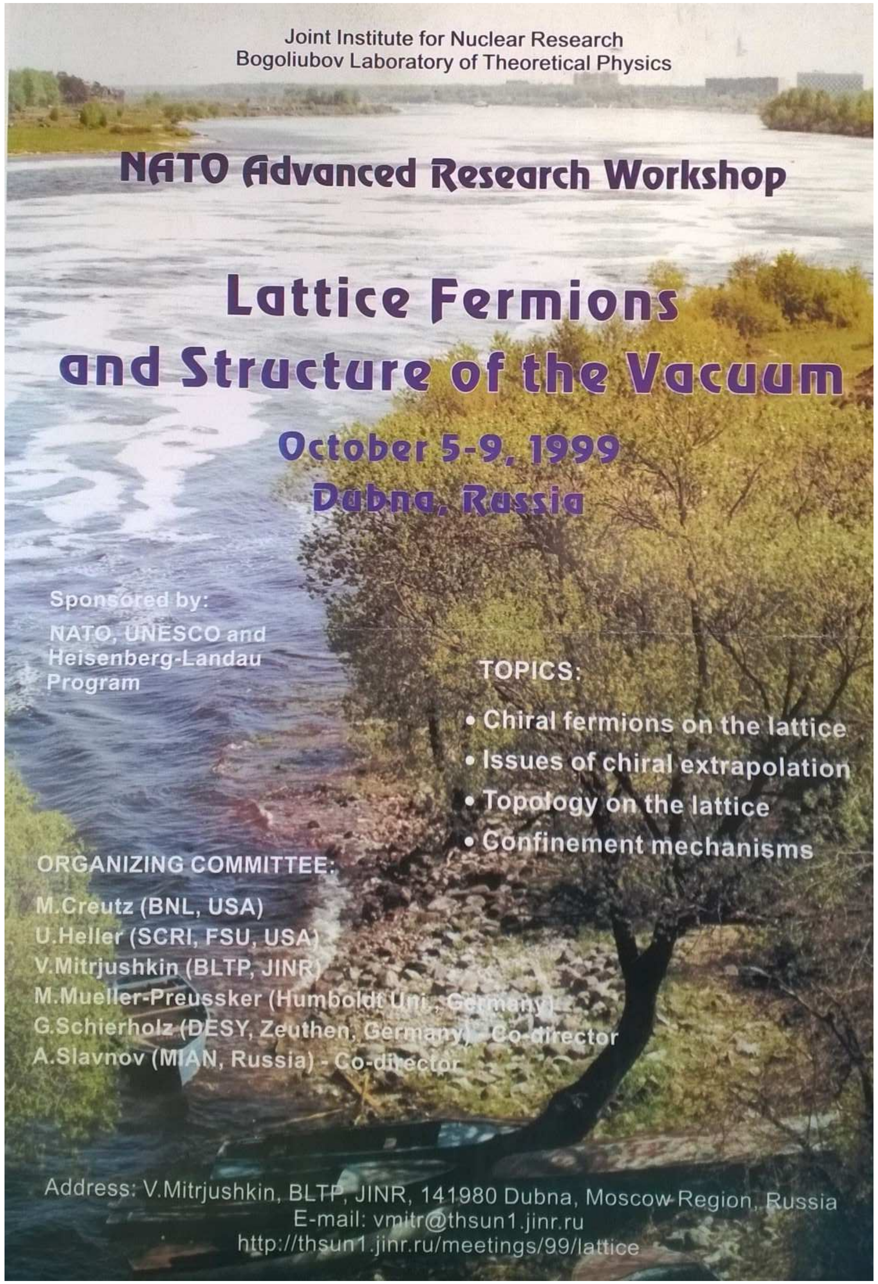
- DESY Hamburg/Zeuthen (G. Schierholz) (constant support to LGT in Russia)
- Niels Bohr Institute Copenhagen (M. L. Laursen)
- University of Amsterdam (A. J. van der Sijs)
- Universität Bielefeld (F. Karsch, J. Engels et al.)
- Universität Wuppertal (K. Schilling)
- Universita degli Studi Pisa (A. DiGiacomo et al.)
- ITEP Moscow (M. I. Polikarpov, A. I. Veselov et al.)
- University of Kanazawa (T. Suzuki)
- University of Leiden (P. van Baal)

Topics under investigation before 2000 :

- **Vacuum structure : instantons, monopoles, gluon condensate**
- **Thermal phase transition and topological aspects**
- **Mean field and Complex Langevin for finite baryonic density**
- **Effective algorithms for fermions**
- **Improved gauge fixing : Maximal Abelian gauge and Landau gauge**

The biggest international event in Dubna in Lattice Field Theory :

"Lattice fermions and structure of the vacuum",
NATO Advanced Research Workshop, Dubna, Russia, October 5-9, 1999,
organised by
V. K. Mitrjushkin (Dubna, JINR) and G. Schierholz (DESY, Zeuthen)


Joint Institute for Nuclear Research
Bogoliubov Laboratory of Theoretical Physics
NATO Advanced Research Workshop
Lattice Fermions
and Structure of the Vacuum
October 5-9, 1999
Dubna, Russia
Sponsored by:
NATO, UNESCO and
Heisenberg-Landau
Program
TOPICS:
• Chiral fermions on the lattice
• Issues of chiral extrapolation
• Topology on the lattice
• Confinement mechanisms
ORGANIZING COMMITTEE:
M.Creutz (BNL, USA)
U.Heller (SCRI, FSU, USA)
V.Mitrjushkin (BLTP, JINR)
M.Mueller-Preussker (Humboldt Uni., Germany)
G.Schierholz (DESY, Zeuthen, Germany) - Co-director
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http://thsun1.jinr.ru/meetings/99/lattice

Информация | Ссылки (0) | Цитирования: (0) | Файлы | Графики

Lattice fermions and structure of the vacuum. Proceedings, NATO Advanced Research Workshop, Dubna, Russia, October 5-9, 1999

V.K. Mitrjushkin (Dubna, JINR), G. Schierholz (DESY, Zeuthen)

2000 - 383 pages

Prepared for NATO Advanced Research Workshop on Lattice Ferm Conference: C99-10-05.3 (Dordrecht, Netherlands: Kluwer Academic (2000) 383 p)
Contributions

Keyword(s): INSPIRE: book | conference: Dubna 1999/10/05 | fermion: lattice field theory | gauge field theory | symmetry: chiral | quantum chromodynamics: vacuum state

Record added 2001-07-25, last modified 2012-11-03

Organisation of the Lattice Summer Schools in Dubna in 2010's

supported by "Helmholtz Association" of German Research Centers

- **Dubna International Advanced School of Theoretical Physics Helmholtz International School "Lattice QCD, Hadron Structure and Hadronic Matter", September 5 - 17, 2011 Organized by R. Sommer (DESY Zeuthen) with A. Sorin and M. Müller-Preussker**
- **Dubna International Advanced School of Theoretical Physics Helmholtz International School "Lattice QCD, Hadron Structure and Hadronic Matter", August 25 to September 6, 2014 Organized by E.-M. Ilgenfritz (JINR) with O. Teryaev and O. Philipsen**

Topics of the Dubna-Humboldt-ITEP-IHEP Lattice Collaboration 2000 – 2013 :

- **Study of ghost and gluon propagators of Yang-Mills Theory and full QCD (IR asymptotics at $T = 0$ and behavior at $T \approx T_c$)**
- **Fighting the Gribov Ambiguity in Landau gauge fixing (clarifying the IR asymptotics at $T = 0$, decoupling vs. scaling solution)**
- **Thermal monopoles near the phase transition (resp. crossover) of Yang-Mills Theory and full QCD**
- **Effect of the thermal phase transitions on gluon and ghost propagators (and recovering the transition from them)**
- **Topological structure near the phase transition (resp. crossover) of Yang-Mills Theory and full QCD (van Baal calorons and dyons)**

Part of this activity was awarded by the JINR Prize 2015
for a cycle of 20 theoretical papers
"Lattice studies of Landau gauge gluon and ghost propagators in Quantum Chromodynamics",
authored by the Russian-German group
I. Bogolubsky, V. Bornyakov, E.-M. Ilgenfritz, V. Mitrjushkin, M. Müller-Preussker and A. Sternbeck, **the most–cited paper is:**
I.L. Bogolubsky et al., Phys.Lett. B676 (2009) 69 → cited by 312 records

After that "chapter was closed", it was "high time" for a re-orientation to optimize our contribution to the physics of the Quark-Gluon-Plasma :
more systematically doing Lattice Thermodynamics, combined with propagator, vertex etc. studies in order to keep contact with Continuum QCD approaches (DSE and FRG, effective potential).

2. Motivating the Running Project : Twisted Mass Approach at $T \neq 0$

What changes are going on in Lattice QCD ? Where should we go to ?

- **Nuclear Physics** gets increasing contribution from lattice. Connecting QCD with nuclear physics via the lattice – a viable option for BLTP ?
- **Spectroscopy** for low energy high precision physics (FAIR $\rightarrow$ PANDA) hadron spectroscopy, exotica, quarkonia, glueballs, hadron structure
- **QGP** : a long way to go towards Lattice QCD at baryonic chemical potential $\mu_B \neq 0$; progress has been recently achieved (complexification, complex Langevin simulation, Lefschetz thimbles, dualization)
- **QGP** : fruitful cooperation with Continuum QCD (SDE and FRG) (helps extending quenched $\rightarrow$ unquenched, extending $\mu = 0 \rightarrow \mu \neq 0$ etc., gives assessments of approximations being made there)

- QGP : new observables became accessible to describe the QGP (jet quenching, transport coefficients, hadronization, real–time extensions (non-equilibrium QCD and kinetics), spectral functions (gluons and quarks), their extraction is non-trivial !)

Thus, in view of these numerous “hot cross-links”,

- conventional hadron gas resp. quark-gluon plasma thermodynamics (however within an unconventional fermion discretization) remains an interesting option, even restricted to zero baryonic density ($\mu_B = 0$).

For this reason,
twisted mass lattice QCD became the topic of first choice for collaboration between Dubna and Humboldt Universität Berlin !

Suitable also for training of new lattice people in and for Dubna !

Two master students, in Dubna University (from Kazakhstan) and in Taras Shevchenko University (in Kiev) have been supervised with this aim to prepare they for work in Lattice QCD.

Results : One successful master thesis (Orinay Baidlaeva) and one successful bachelor degree (Oleksii Grinyuk), before they both got lost, unfortunately.

Series of lectures in BLTP in Kiev in 2013.

One Ukrainian colleague (Maksym Teslyk) was member of my sector No. 17 in BLTP (before he left Dubna because of the Russian-Ukrainian tensions). I had introduced him into real-time lattice techniques.

Finally, one new collaborator in Dubna : Anton Trunin (PhD 2014)

Two publications resulting from his contributions to the "Strangeness in Quark Matter" Conference, Dubna 2015 :

1. **Topological susceptibility from $N_f = 2 + 1 + 1$ lattice QCD at nonzero temperature,**
   A. Trunin, F. Burger, E.-M. Ilgenfritz, M. P. Lombardo, M. Müller-Preussker,
   J. Phys. Conf. Ser. 668 (2016) no.1, 012123 (arXiv:1510.02265)

2. **Towards the quark–gluon plasma Equation of State with dynamical strange and charm quarks,**
   F. Burger, E.-M. Ilgenfritz, M. P. Lombardo, M. Müller-Preussker, A. Trunin,
   J. Phys. Conf. Ser. 668 (2016) no.1, 012092 (arXiv:1510.02262)

Now we are facing a dramatic situation, but new options are opening :

- **Closing of the lattice group of M. Müller-Preussker in HU Berlin.**
- **Huge lattice ensembles remaining from simulations, which are only partly evaluated (have to be moved from HLRN to CINECA [Italy]).**
- **New partners from other German Universities are interested to use the data, at least from the point of view of Continuum QCD (Heidelberg and Giessen).**
- **New partners from other German Universities are considering now to become more active (or, active again !) "on lattice" with manpower, master students and hardware (Giessen and Jena (A. Sternbeck), eventually Heidelberg).**

A good opportunity to continue working on Lattice QCD in Dubna !

Physics–wise : Current situation in Lattice Thermodynamics :

- Thermodynamic simulations mostly done with staggered fermions : Computationally least demanding ! Has remnant of chiral symmetry. Many improvements made since the advent of staggered fermions : p4, asqtad, stout $\rightarrow$ Highly Improved Staggered Quarks (HISQ).
- They had to fight with problems like taste symmetry breaking.
- Other problems remaining : rooting ? locality of the action ?
- Comparison/parallel studies with other fermion discretization schemes are an obvious "must" :

1. Wilson fermions (have become relatively popular recently)
2. Domain wall fermions (still difficult, a bit "exclusive")
3. Overlap fermions (still difficult, expensive, "perfect")

Improvement in a necessary. Two methods exist for Wilson fermions :

1. **Clover improvement, by adding a Pauli term w and w/o stout smearing (CP-PACS-Coll., WHOT-QCD-Coll., DESY-ITEP-Kanazawa-Coll.)**
2. **Twisted mass improvement (European Twisted Mass Coll., concentrating on $T = 0$ hadron physics)**

The latter improvement scheme has been applied to finite temperature only by few people, the tmfT Collaboration (proposed at LATTICE 2006).

The Wilson twisted mass fermion action is written for a doublet in a twisted basis $\bar{\psi}, \psi$ (τ^3 in flavor space):

$$S_F[\psi\,,\bar{\psi}\,,U] = a^4 \sum_x \left[\bar{\psi}(x)\left(1 + \kappa D_W[U] + i\kappa\mu_l\gamma_5\tau^3\right)\psi(x)\right]$$

where $D_W[U] = \gamma_\mu \frac{1}{2a}\left(\nabla_\mu + \nabla^*_\mu\right) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu$ is the Wilson-Dirac operator.

Describes a minimum of $N_f = 2$ light quarks : μ_l regulates the light sector.

Use of this fermion action is made in conjunction with a tree-level Symanzik improved gauge action : public code available, see https://github.com/etmc/tmLQCD/

$$S_G[U] = \beta \left[c_0 \sum_P \left(1 - \frac{1}{3} \mathbf{Re} \left(\mathrm{tr} \left[U(P) \right] \right) \right) + c_1 \sum_R \left(1 - \frac{1}{3} \mathbf{Re} \left(\mathrm{tr} \left[U(R) \right] \right) \right) \right]$$

where $\beta = 6/g_0^2$ **and** $U(P)$, $U(R)$ **are plaquette and rectangle loops.**

With weight coefficients $c_0 + 8\,c_1 = 1$, **and** $c_1 = -1/12$

This choice corresponds to the conventions adopted by European Twisted Mass Collaboration (ETMC) for the hadron sector at $T = 0$ **with** $N_f = 2$ **(referred to for calibration of our** $T \neq 0$ **lattices).**

Main advantages of the twisted mass approach :

- prevents exceptional configurations which are spoiling, for example, clover improved Wilson fermion simulations
- twisted mass provides a natural infrared cutoff (the twisted mass μ_l)
- at maximal twist, with κ set to $\kappa_c(\beta)$, the twisted mass term μ_l takes the role of the mass term, while automatic $O(a)$ improvement is guaranteed.

This attractive feature has been observed first by
R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007

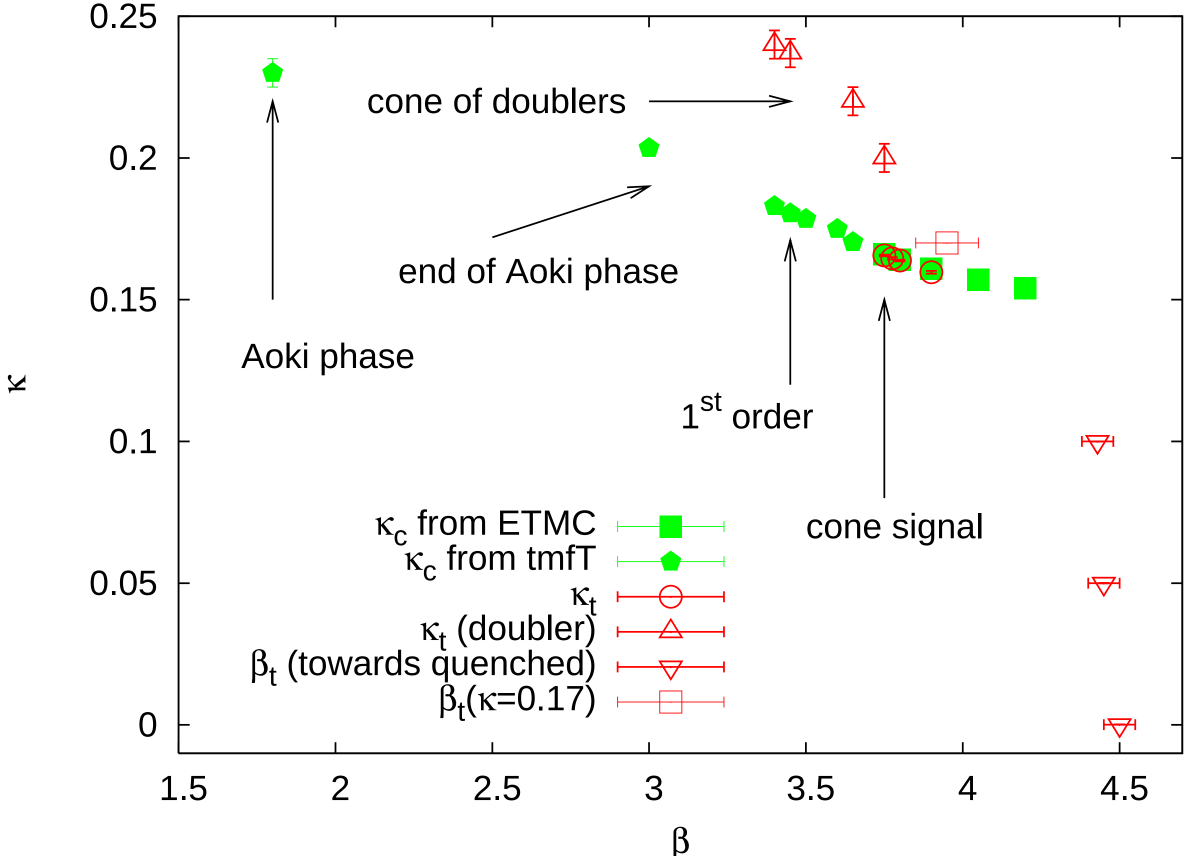
Disadvantage :

- explicit flavor symmetry breaking, which is a small remnant of the "Aoki phase" : scale setting is necessary with $m_{\pi^\pm}$ instead of m_{π^0}.

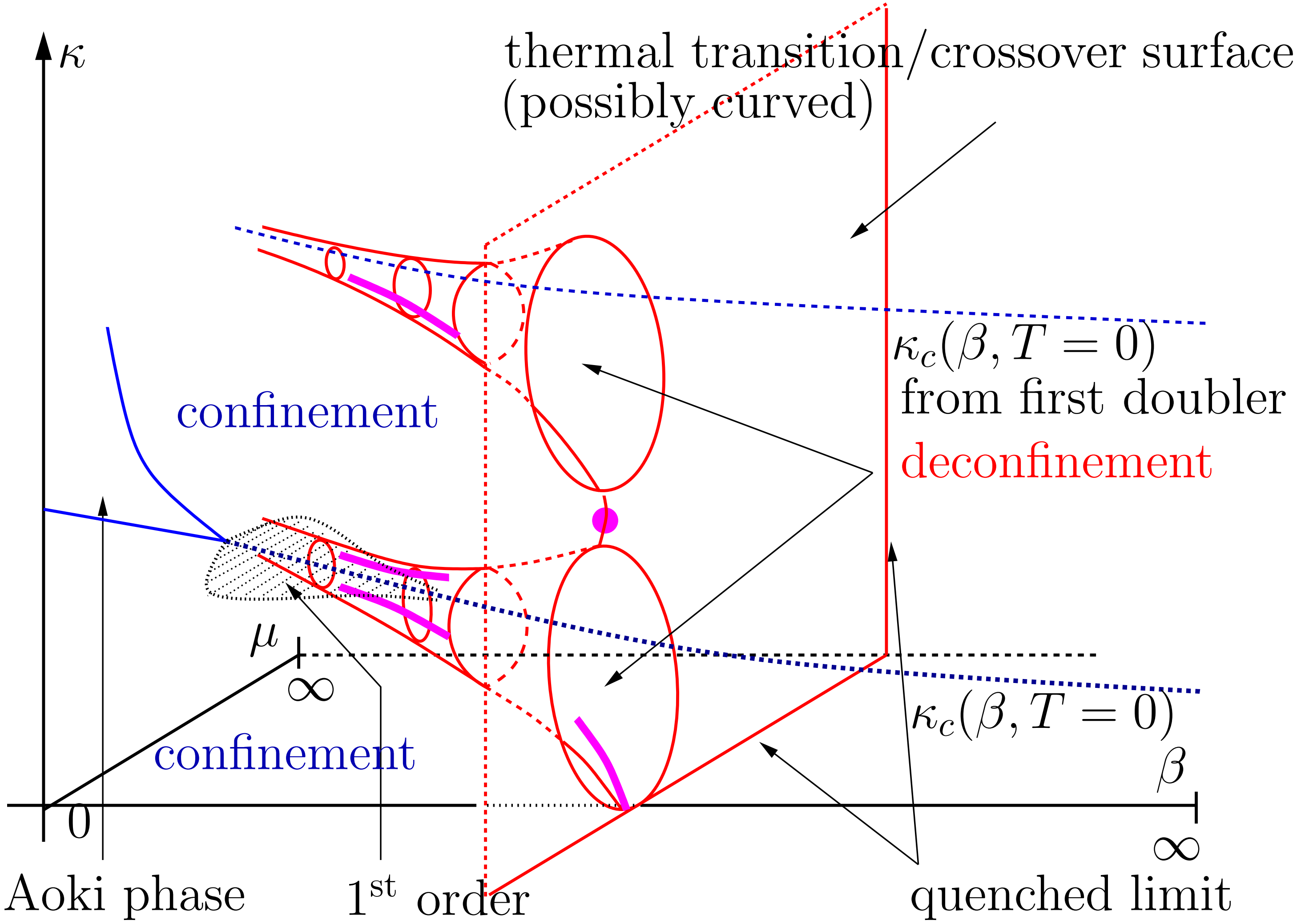
Our first milestone. A global overview of phase structure in :

"Phase structure of thermal lattice QCD with $N_f = 2$ twisted mass Wilson fermions", Phys. Rev. D 80 (2009) 094502

E.-M. I., K. Jansen, M.P. Lombardo, M. Müller-Preussker, M. Petschlies, O. Philipsen, L. Zeidlewicz (projected on κ-β plane)



Observations for $N_f = 2$ collected in a more suggestive 3-dimensional phase diagram (illustrating the "cone conjecture" by M. Creutz)



What is the physically relevant branch ?

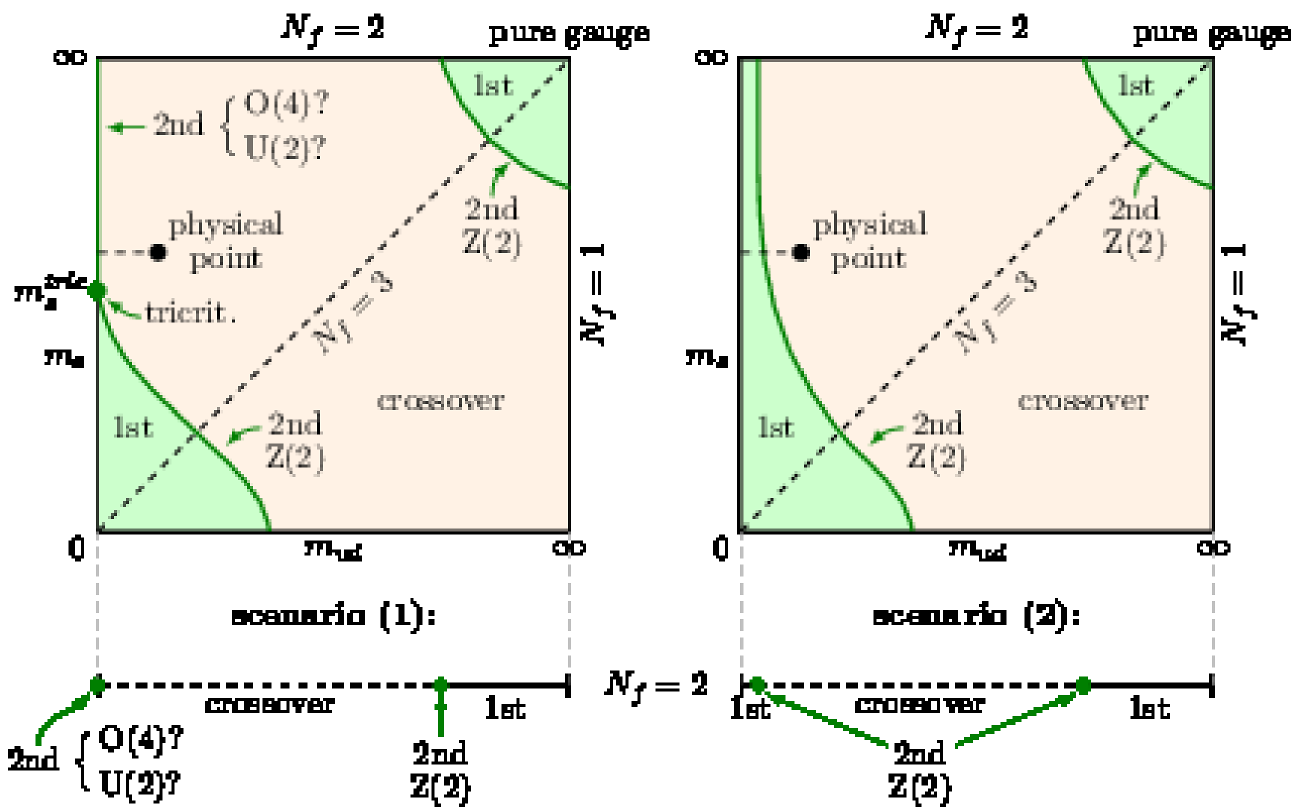
- **Only the lower cone is connected with the quenched limit !**
  Varying the quark mass $\rightarrow \infty$, a first a critical endpoint is passed, at which the crossover goes over into a first order transition.
  The first order line finally ends in the quenched endpoint at $\kappa = 0$.

- **How does a line of constant physics (LCP) intersect Creutz' cone ?**
  - **LCP has to run at maximal twist ! Tuning $\kappa = \kappa_c(\beta)$ is required.**
  - **LCP must not run at $\mu_l = \text{ const}$! Tuning $\mu_l = \mu_0(\beta)$ is required.**

The analysis rests on calibration simulations made at $T = 0$,

- **done by the ETM Collaboration**
- **additional $T = 0$ simulations by the tmfT Collaboration**

Quark mass dependence of the Nature of the thermal phase transition with three flavours (two scenarios, in particular for $N_f = 2$, upper rim)



The Columbia Plot

Chiral limit ? Simulations at unphysically massive light quarks !

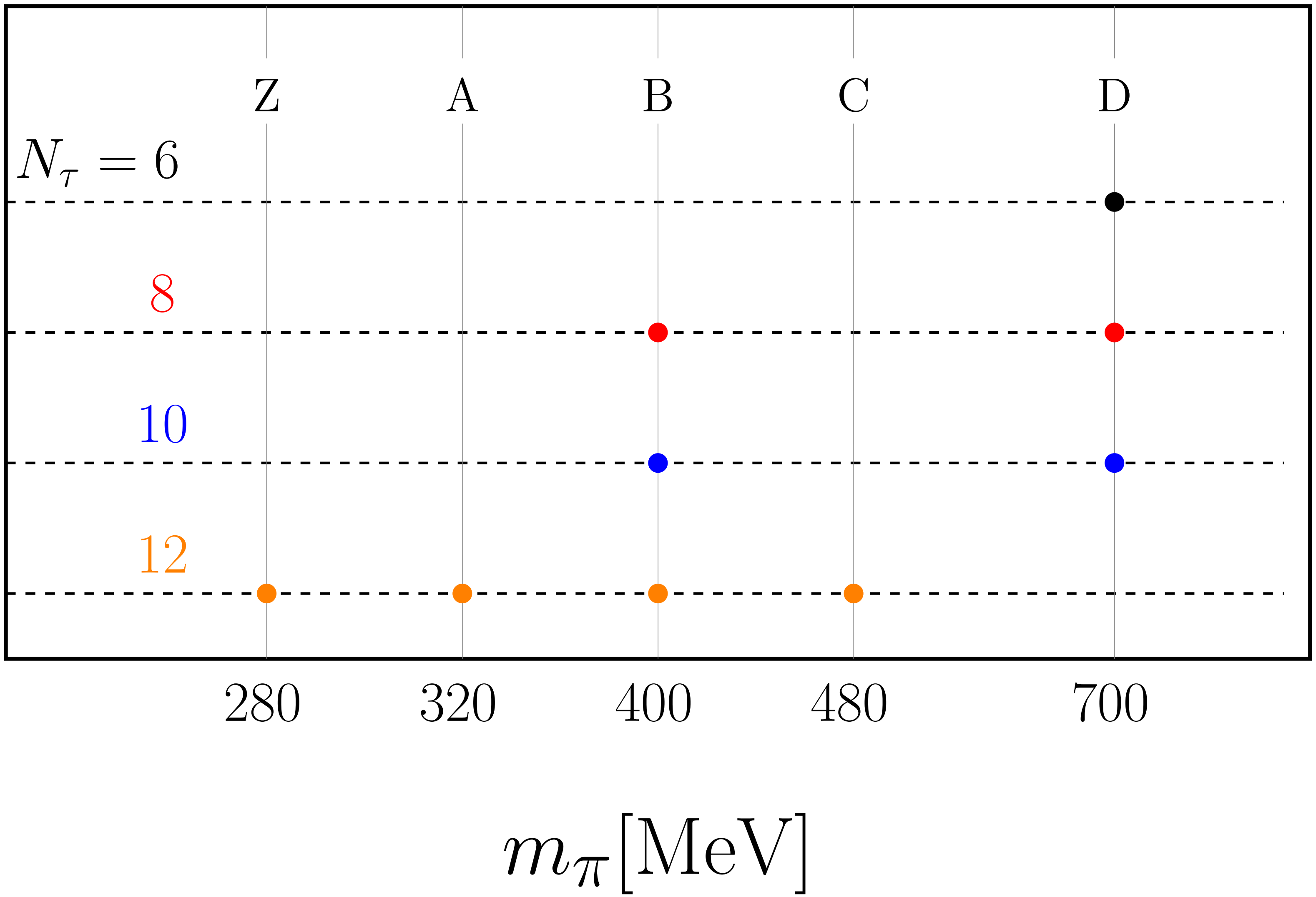
- For natural quark masses : still crossover instead of phase transition.
- Nature of the phase transition with $N_f = 2$ flavors in the chiral limit is of high theoretical interest (not settled up to now), it is depending how the $U_A(1)$ symmetry is restored (B. B. Brandt, H. Meyer et al., arXiv:1608.06882).
- Does “our QCD” belong to the $Z(2)$ or $O(4)$ equivalence classes ?
- Majority of previous results for $N_f = 2$ compatible with $O(4)$.

Next step ? How can we simulate more flavors (partly natural masses) ?

- Rotation in flavor space requires doublets : one degenerate for $N_f = 2$
- Two doublets (light degenerate, heavy non-degenerate) : $N_f = 2 + 1 + 1$

3. Simulation Setup to Find the Crossover Temperatures for $N_f = 2$

- **Evaluated : four families of ensembles : A, B, C, Z (referring to $\pi^{\pm}$ mass)**
- **A12 etc. referring to N_τ (number of time slices, fine or coarse lattice)**
- **populate the three-dimensional phase diagram (β, κ, μ_0)**
- **a β scan should fix the position of the crossover line**
- **maximal twist: requires tuning of $\kappa = \kappa_c(T = 0, \beta)$**
- **fixed $m_{\pi^{\pm}}$: requires tuning of $a\mu_l = a\mu_0(\beta) = C\exp\left(-\beta/(12\beta_0)\right)$ (obtained from a one-loop fit or a two-loop fit)**
- **such fits for various families of $T = 0$ simulations are based on data of the ETM-Collaboration [published in JHEP 08 097 (2010)]**


Z
A
B
C
D
$N_\tau = 6$
8
10
12
280
320
400
480
700
m_π[MeV]

List of β-scans (here only for smallest a, $N_\tau = 12$)

- **A12:** $32^3 \times 12$, $3.84 \leq \beta \leq 3.99$,
  $m_\pi = 316(16)$ **MeV**, $r_0 m_\pi = 0.673(42)$
  $\beta_\chi = 3.89(3)$,
  $T_\chi = 202(7)$ **MeV**
- **B12:** $32^3 \times 12$, $3.86 \leq \beta \leq 4.35$,
  $m_\pi = 398(20)$ **MeV**, $r_0 m_\pi = 0.847(53)$
  $\beta_\chi = 3.93(2)$, $\beta_{\text{deconf}} = 4.027(14)$,
  $T_\chi = 217(5)$ **MeV**, $T_{\text{deconf}} = 249(5)$ **MeV**
- **C12:** $32^3 \times 12$, $3.90 \leq \beta \leq 4.07$,
  $m_\pi = 469(24)$ **MeV**, $r_0 m_\pi = 0.998(62)$
  $\beta_\chi = 3.97(3)$, $\beta_{\text{deconf}} = 4.050(15)$,
  $T_\chi = 229(5)$ **MeV**, $T_{\text{deconf}} = 258(5)$ **MeV**

One T_χ or two temperatures T_χ and $T_{\rm deconf}$ (chiral and deconfining) are localized by considering ($T_\chi < T_{\rm deconf}$) :

- **chiral susceptibility**

$$\chi_{\bar\psi\psi} = \frac{\partial\langle\bar\psi\psi\rangle}{\partial m_q}$$

- **disconnected part of it (looking for a Gaussian peak)**

$$\sigma^2_{\bar\psi\psi} = \frac{V}{T}\left(\langle(\bar\psi\psi)^2\rangle - \langle\bar\psi\psi\rangle^2\right)$$

with $\langle\bar\psi\psi\rangle$ evaluated as a stochastic estimator

- **renormalized (subtracted) chiral condensate**

$$R_{\langle\bar\psi\psi\rangle} = \frac{\langle\bar\psi\psi\rangle(T,\mu_l) - \langle\bar\psi\psi\rangle(0,\mu_l) + \langle\bar\psi\psi\rangle(0,0)}{\langle\bar\psi\psi\rangle(0,0)}$$

- **renormalized Polyakov loop (searching for an inflection point)**

$$\langle \mathrm{Re}(L) \rangle_R = \langle \mathrm{Re}(L) \rangle_{\mathrm{bare}} \exp\left(V(r_0)/2T\right)$$

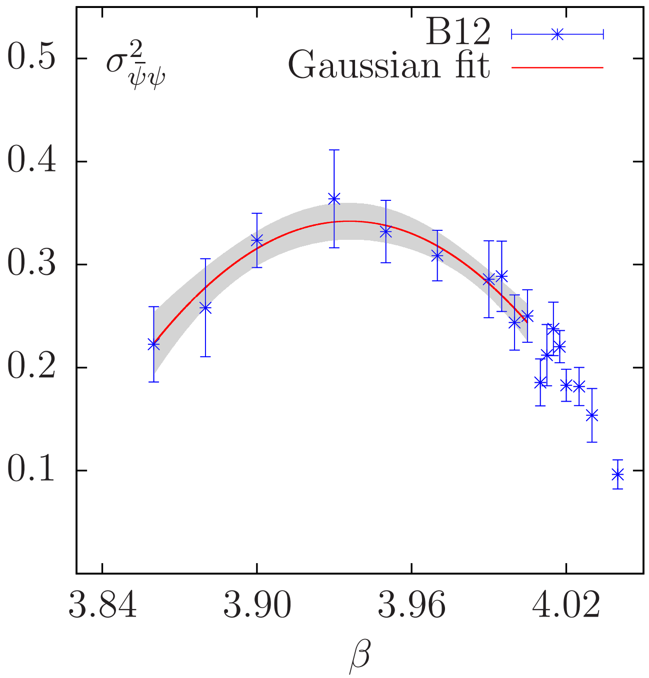
- **and its susceptibility (searching for a Gaussian peak)**

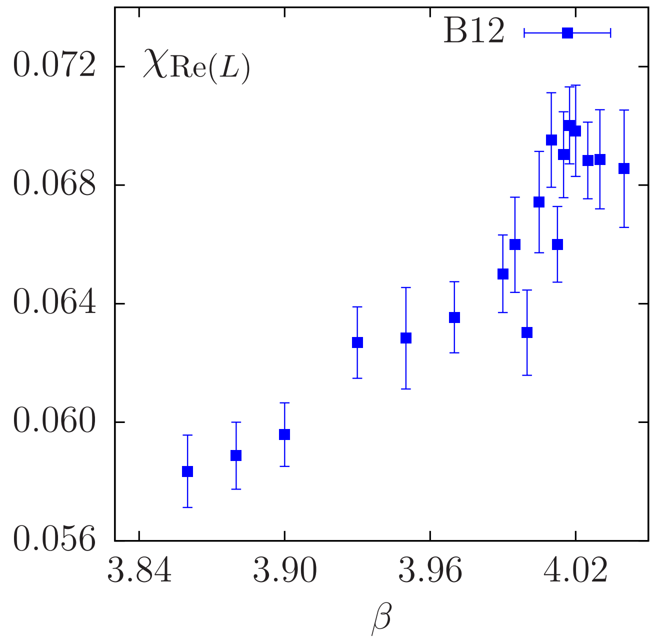
$$\chi_{\mathrm{Re}(L)}$$

The m_π-dependence and chiral extrapolations are discussed in papers :

- **arXiv:1102.4530v2 (finally Phys. Rev. D 87 (2013) 074508)**
- **arXiv:1212.0982 (F. Burger et al., at LATTICE 2012)**

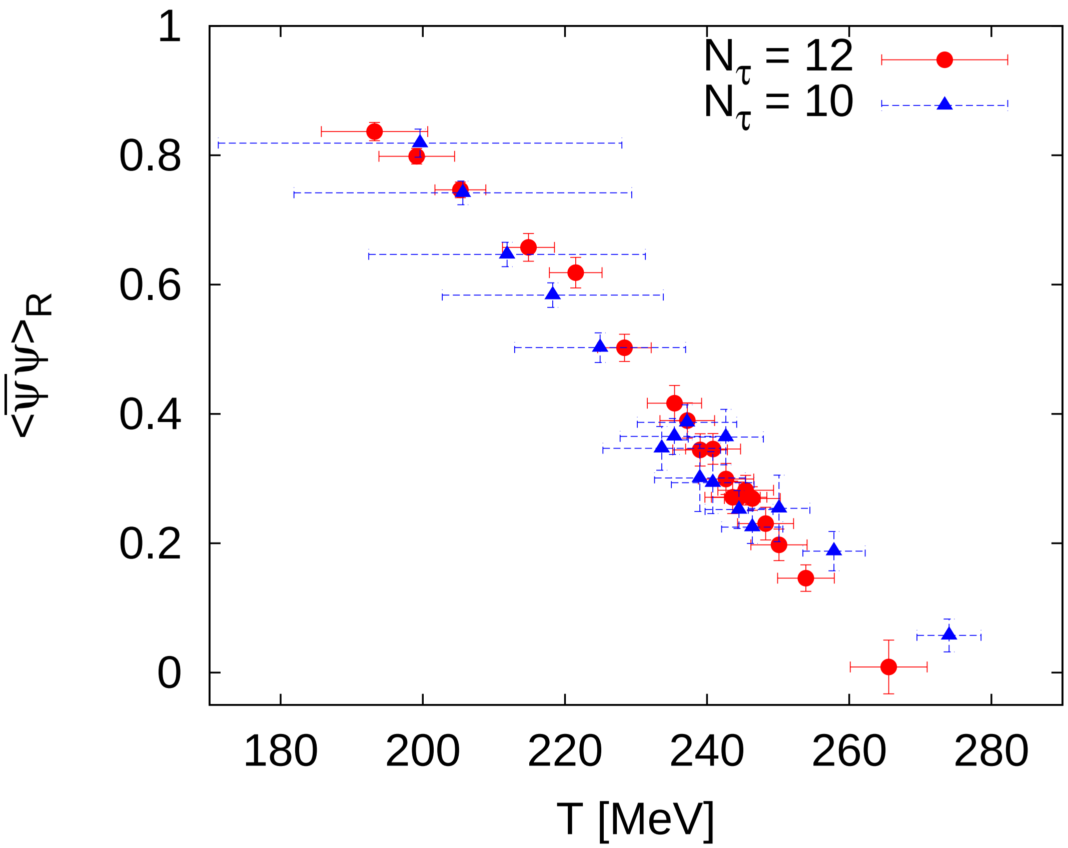
Chiral susceptibility and Polyakov loop susceptibility for B12



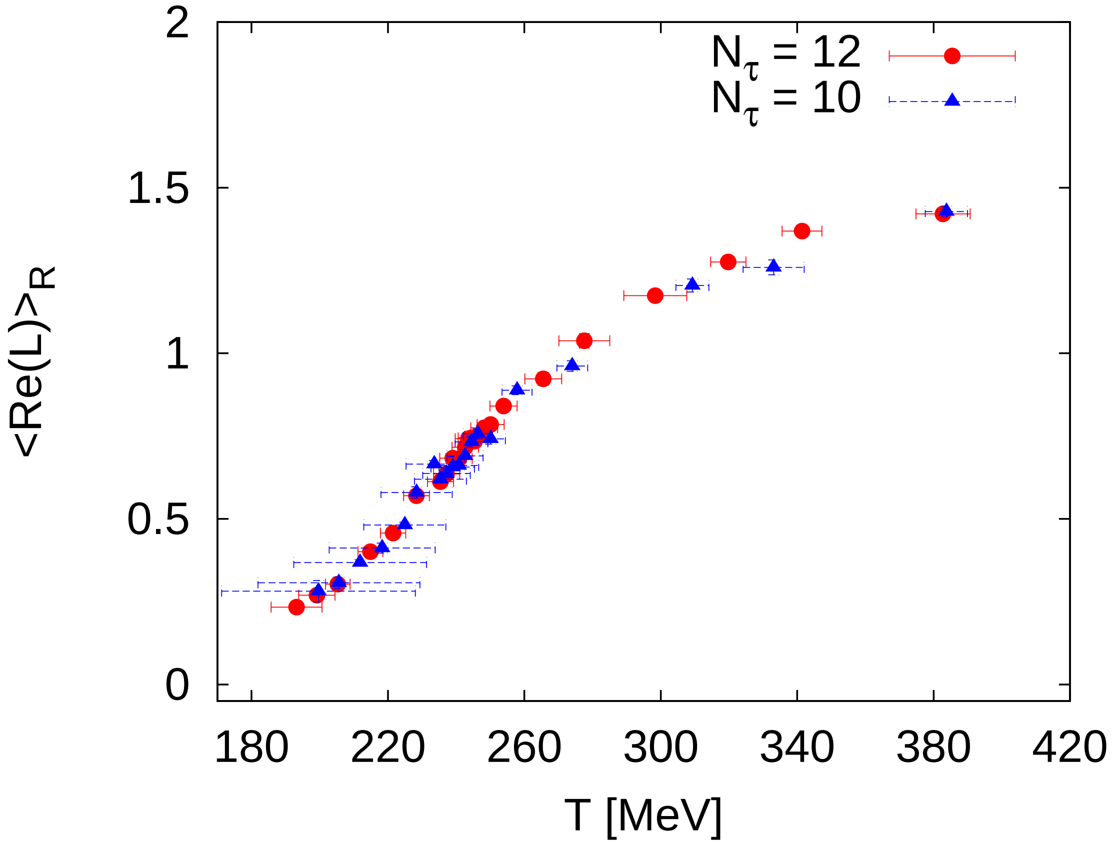


Renormalized $\langle\bar{\psi}\psi\rangle$ for B12 and B10

(with N_τ dependence successfully removed)



Renormalized Polyakov loop $\langle \mathrm{Re}(L) \rangle_R$ for B12 and B10
(with N_τ dependence successfully removed)



4. Towards the Chiral Limit of T_χ for $N_f = 2$

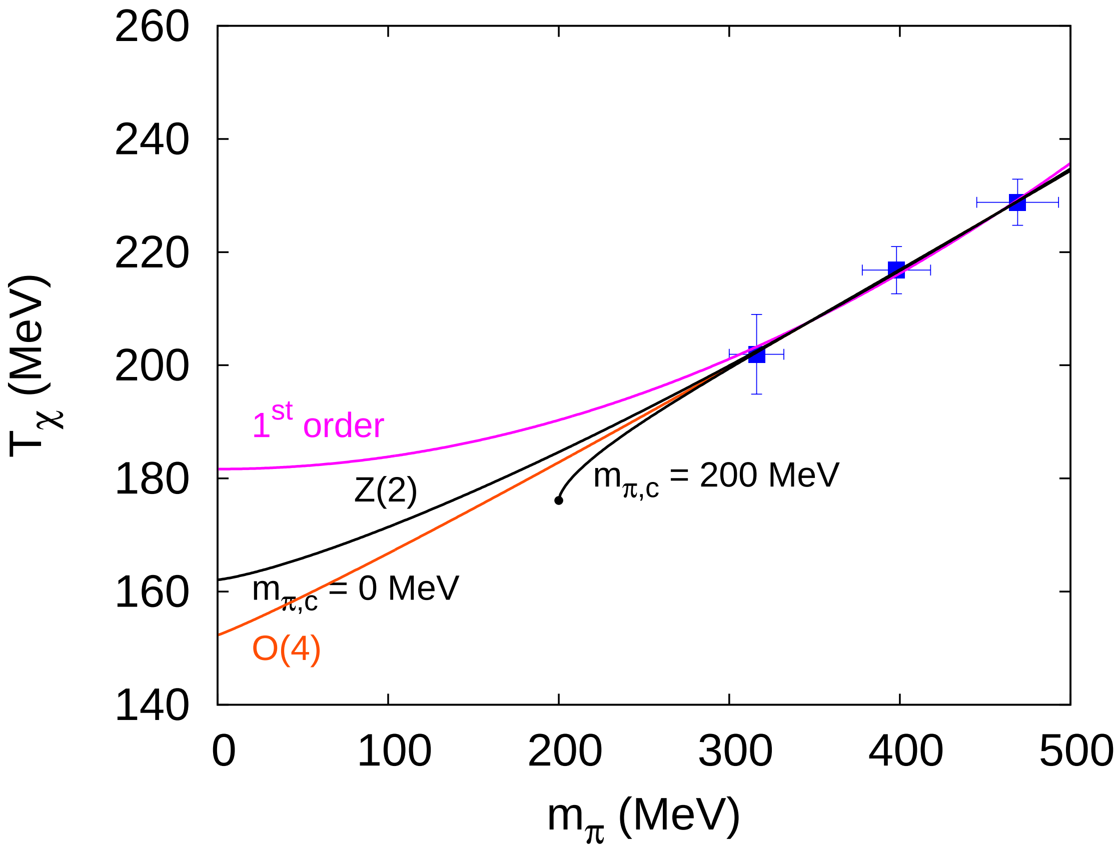
Chiral extrapolations for $T_\chi(m_\pi)$ for various scenarios (from χPT)

$$T_\chi(m_\pi) = T_\chi(m_\pi = 0) + A \; m_\pi^{2/(\tilde{\beta}\delta)}$$

with critical indices $\tilde{\beta}$, δ corresponding to the respective equivalence classes of three-dimensional spin models : $O(4)$ or $Z(2)$ (Ising)

- $O(4)$ **:** $2/(\tilde{\beta}\delta) = 1.08$ **(close to linear fit) leads to** $T_\chi(m_\pi = 0) = 152(26)$ **MeV**
- $Z(2)$ **: two cases** $m_{\pi,c} = 0$ **or** $m_{\pi,c} \neq 0$**;**
  these lead to $T_\chi(m_\pi \to 0)$ **between** $O(4)$ **and 1-st order scenario**
- **first order : in literature one takes formally** $2/(\tilde{\beta}\delta) = 2$**; the quadratic fit leads to** $T_\chi(m_\pi = 0) = 182(14)$ **MeV**
  (applicability of these "critical indices" questionable in this context)

Chiral extrapolations for $T_\chi(m_\pi)$ for various scenarios



Summary : scenarios for the $N_f = 2$ chiral limit

1. **first order transition:**
   fit gives $T_\chi(m_\pi = 0) = 182(14)$ **MeV**
2. $O(4)$ **second order transition down to** $m_\pi = 0$ **:**
   fit gives $T_\chi(m_\pi = 0) = 152(26)$ **MeV**
3. **second order** $Z(2)$ **transition, fits give** $T_\chi(m_\pi \to 0)$ **in between.**
   - **either with a critical point separating second order** $(m_\pi > m_{\pi,c})$ **from first order transition (for** $m_\pi < m_{\pi,c}$**) with** $m_{\pi,c} \neq 0$
   - **or as second order transition down to** $m_{\pi,c} = 0$ **: leading to** $T_\chi(m_\pi = 0) = 162(16)$ **MeV .**

The Z12 results ($m_\pi \approx 280$ **MeV) was hoped to exclude the first order scenario ! This was not achieved by tmfT before** $N_f = 2$ **was abandoned.**

5. Simulation Setup for $N_f = 2 + 1 + 1$ flavors

Our aims :

- Exploring QCD thermodynamics with realistic strange and charm quark masses, with $\pi^{\pm}$ masses from approx. 210 to 470 MeV
- Assessing the influence of heavy flavors on the crossover behavior
- Fixing the contribution of strange and charm quarks to the Equation of State within a T interval from 150 to 600 MeV
- For this purpose, making use of the twisted mass approach for two doublets (a light and a heavy) of Wilson fermions
- Instead of locating the transition by β-scans (as we did for $N_f = 2$), we are now making use of the fixed-scale approach (fixing β)

- **The approach to the continuum limit is taken under control by simulating for a set of β-values (lattice spacings)**
- **This philosophy allows to make contact to $T = 0$ results of the ETM Collaboration ($T = 0$ vacuum expectation values, masses vs. bare parameters, β-functions and anomalous dimensions from a global fitting)**

To describe $2+1+1$ flavors, one adds a second, non-degenerate doublet of heavy h to the light, degenerate doublet l :

(τ^3 and τ^1 in flavor space)

$$
\begin{aligned}
S_F[\psi\,,\bar{\psi}\,,U] \;=\;& a^4 \sum_x \left[\bar{\psi}_l(x)\left(1-\kappa D_W[U] + i2\kappa a\mu_l\gamma_5\tau^3\right)\psi_l(x)\right] \\
+\;& a^4 \sum_x \left[\bar{\psi}_h(x)\left(1-\kappa D_W[U] + i2\kappa a\mu_\sigma\gamma_5\tau^1 + 2\kappa a\mu_\delta\tau^3\right)\psi_h(x)\right]
\end{aligned}
$$

where the renormalized s and c quark masses are related by

$$
(m_{s,c})_R = \frac{1}{Z_P}\left(\mu_\sigma \mp \frac{Z_P}{Z_S}\mu_\delta\right)
$$

The Iwasaki action is used now in this context for the gauge sector :

(with $c_0 = 3.648$ and $c_1 = -0.331$)

$$
S_g[U] = \beta\left(c_0 \sum_P [1 - \frac{1}{3}\mathrm{ReTr}\,(U_P)] + c_1 \sum_R [1 - \frac{1}{3}\mathrm{ReTr}\,(U_R)]\right).
$$

How is the simulation done ?

- **Standard Hybrid Monte Carlo simulation is used for the two doublets instead of one (two fermionic forces in Hamiltonian dynamics).**
- **Modern HMC with all tricks. C. Urbach and K. Jansen, "tmLQCD: A Program suite to simulate Wilson Twisted mass Lattice QCD", Comput. Phys. Commun. 180 (2009) 2717, arXiv:0905.3331**
- **The hopping parameter $\kappa_l = \kappa_c(\beta)$ must be tuned to maximal twist.**
- **The "light" twisted-mass parameter μ_l characterizes now the (eventually non-physical, however constant) $\pi^{\pm}$-mass.**
- **The two "heavy" twisted-mass parameters, μ_σ and μ_δ, are tuned to the physical point (K and D masses).**

So far we have generated finite temperature configurations for eight sets of parameters corresponding to

- **four values of the charged pion mass of about 470, 370, 260 and 210 MeV for which ...**
- **two, three, two and one values of the lattice spacing (of β) have been considered, respectively : nommenclature relates A, B, D to**

$$a(A) \approx 0.09 \text{ fm} > a(B) \approx 0.08 \text{ fm} > a(D) \approx 0.06 \text{ fm}$$

- **The corresponding lattice spacings have been taken from the paper arXiv: 1406.4310, C. Alexandrou et al. (2014), where they are fixed by comparison with the nucleon sector.**

our	ETMC	$m_\pi^\pm$ [MeV]	a [fm]	$N_\tau \times N_\sigma^3$	statistics
D210	**D15.48**	**213(9)**	**0.0646**	$\{4,6,8,10,12,14,16,18,20,24\} \times 48^3$	**1k-7k**
A260	**A30.32**	**261(11)**	**0.0936**	$\{4,6,8,10,11,12,14\} \times 32^3$	**1k-5k**
				$\{16\} \times 40^3$	**3k**
				$\{20\} \times 48^3$	**4k**
B260	**B25.32**	**256(12)**	**0.0823**	$\{4,5,6,8,10,12,14,16,18\} \times 40^3$	**1k-8k**
A370	**A60.24**	**364(15)**	**0.0936**	$\{3,4,5,6,7,8,9,10,11,12\} \times 24^3$	**2k-9k**
				$\{13,14\} \times 32^3$	**5k,27k**
B370	**B55.32**	**372(17)**	**0.0823**	$\{3,4,5,6,7,8,10,11,12,13,14,15,16\} \times 32^3$	**2k-10k**
B370.24	**B55.32**	**372(17)**	**0.0823**	$\{4,6,8,10,11,12\} \times 24^3$	**3k-10k**
D370	**D45.32**	**369(15)**	**0.0646**	$\{5,6,7,8,9,10,12,14,16\} \times 32^3$	**1k-12k**
				$\{18\} \times 40^3$	**10k**
				$\{20\} \times 48^3$	**10k**
A470	**A100.24s**	**466(19)**	**0.0936**	$\{4,5,6,7,8,9,10,11,12\} \times 24^3$	**3k-8k**
				$\{14\} \times 32^3$	**8k**
B470	**B85.24**	**465(21)**	**0.0823**	$\{4,5,6,7,8,9,10,11,12\} \times 24^3$	**2k-4k**
				$\{13,14\} \times 32^3$	**2.5k,7k**

$T = 0$ base ensembles of ETMC with the respective charged pion masses and lattice spacings a, and a list of finite temperature (i.e. various N_τ) ensembles generated so far by tmfT with the given (typical) statistics.

The temperature is assigned to the ensembles by $1/T = aN_\tau$.

A new way to renormalize the “light” chiral condensate :

A new possibility appears to define a renormalized (subtracted) chiral condensate when a third (heavier) flavor is included (s denotes “strange”).

$$\Delta_{l,s} = \frac{\langle\overline{\psi}\psi\rangle_l - \frac{\mu_l}{\mu_s}\langle\overline{\psi}\psi\rangle_s}{\langle\overline{\psi}\psi\rangle_l^{T=0} - \frac{\mu_l}{\mu_s}\langle\overline{\psi}\psi\rangle_s^{T=0}}\,.$$

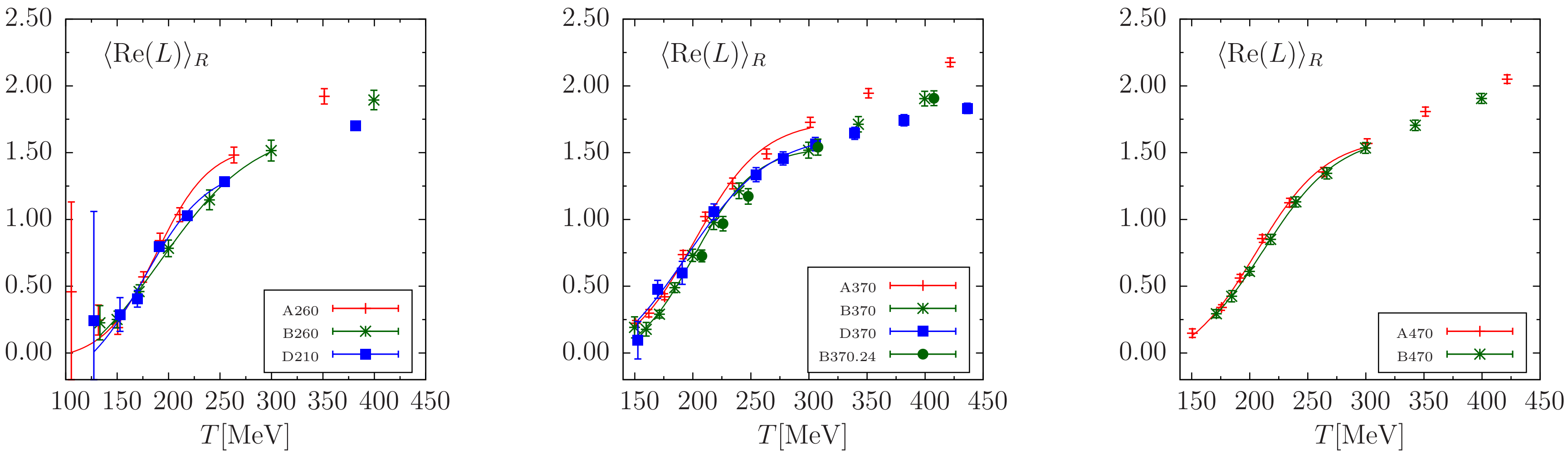
This implies subtraction of a divergent part (proportional to the quark mass, here known from the ratio strange quark condensate/strange quark mass).

Determination of the pseudo-critical temperature (confinement)

from the renormalized Polyakov loop $\langle \mathrm{Re}(L) \rangle_R(T)$

This requires again a measurement of $V(r_0)$!

The static quark-antiquark potential $V(r)$ has been evaluated using APE smearing. For the Sommer scale $r_0 \approx 0.5$ fm we use the values determined by ETMC, for the respective parameter β.



The renormalized Polyakov loop.

Left: **for** $m_\pi = 210$ **MeV (blue points) and for** $m_\pi = 260$ **MeV.**

Middle: **for** $m_\pi = 370$ **MeV. The data of the finite size test ensemble B370.24 is shown slightly shifted to ease reading the figure (no volume effect seen !).** **Right:** **for** $m_\pi = 470$ **MeV.**

The deconfinement transition temperature T_{deconf} is read off as inflection point of a hyperbolic tangent function fit to the renormalized Polyakov loop data

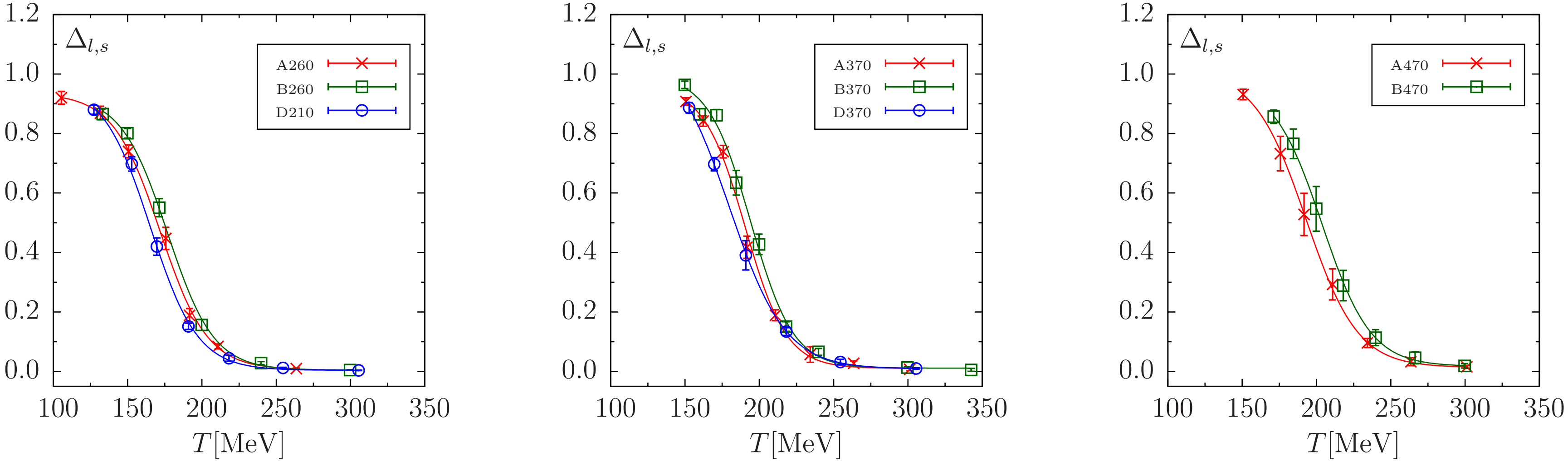
$$\langle \text{Re}(L) \rangle_R = A_P + B_P \tanh\left(C_P(T - T_{\text{deconf}})\right) .$$

The data becomes more and more noisy for larger N_τ which mostly reduces the data quality for the small mass points.

Fits restricted to $T < 310$ MeV.

Determination of the pseudo-critical temperature (chiral transition)

from the renormalized (subtracted) chiral condensate $\Delta_{ls}(T)$



The renormalized (subtracted) chiral condensate Δ_{ls}.

Left: for $m_\pi = 210$ MeV (blue points) and for $m_\pi = 260$ MeV.

Middle: for $m_\pi = 370$ MeV. Right: for $m_\pi = 470$ MeV

The chiral transition temperature T_Δ is read off as the inflection point of a hyperbolic tangent function fit to the renormalized (subtracted) chiral condensate data

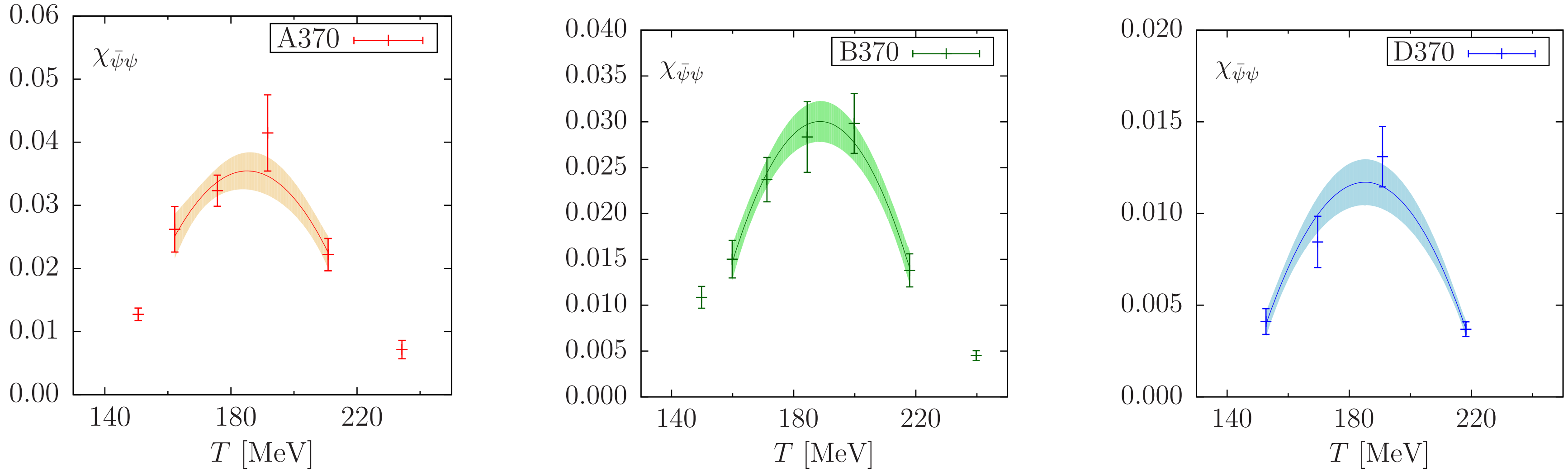
$$\Delta_{ls}(T) = A_\Delta + B_\Delta \tanh\left(-C_\Delta(T - T_\Delta)\right) \ .$$

We always used all available data at low temperatures and used two upper limits for the fit ranges. The main fits were obtained with data with $T < 350$ MeV.

Another fit has been done with $T < 450$ MeV. Half the deviation of the latter from the main fit results was used to obtain the systematic error.

An alternative determination of the pseudo-critical (chiral) temperature

from the chiral susceptibility $\chi_{\bar{\psi}\psi}(T)$ (as practized before for $N_f = 2$)



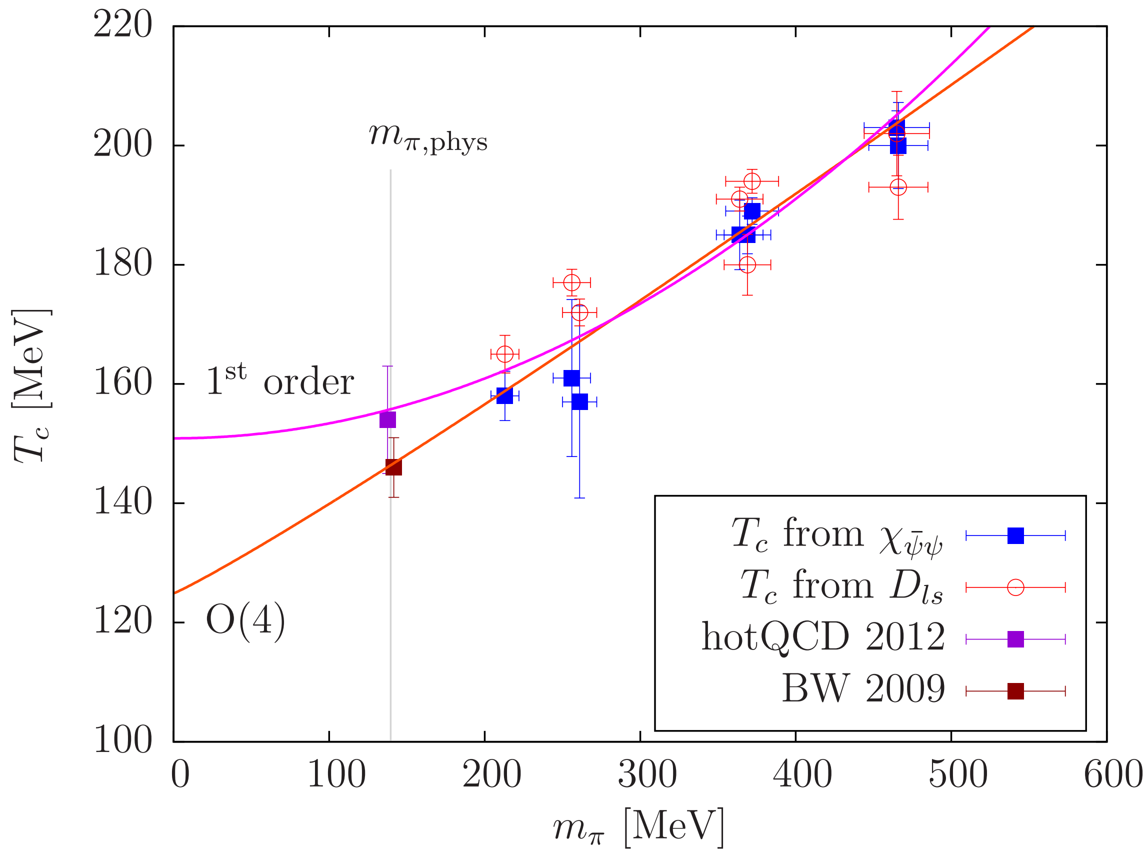
The bare (disconnected) chiral susceptibility $\chi_{\bar{\psi}\psi}$ and fits for different ensembles. Left: A370, middle: B370, Right: D370 (data are shown here only for $m_\pi \approx 370$ MeV).

The chiral transition temperature T_χ is obtained from fits with the parabolic ansatz (no model) for $\chi_{\bar{\psi}\psi}(T)$

$$\chi_{\bar{\psi}\psi}(T) = A_\chi + B_\chi \left(T - T_\chi\right)^2 \ .$$

Ensemble	a [fm]	m_π [MeV]	T_χ [MeV]	T_Δ [MeV]	T_{deconf} [MeV]
D210	0.065	213	158(1)(4)	165(3)(1)	176(8)(8)
A260	0.094	261	157(8)(14)	172(2)(1)	188(6)(1)
B260	0.082	256	161(13)(2)	177(2)(1)	192(9)(2)
A370	0.094	364	185(5)(3)	191(2)(0)	202(3)(0)
B370	0.082	372	189(2)(1)	194(2)(0)	201(6)(0)
D370	0.065	369	185(1)(3)	180(5)(1)	193(13)(2)
A470	0.094	466	200(4)(6)	193(5)(2)	205(4)(2)
B470	0.082	465	203(2)(2)	202(7)(1)	212(6)(1)

Summary of fit-estimated pseudo-critical temperatures using fermionic and gluonic observables. (unpublished !)



Chiral extrapolation and extrapolation to the physical pion mass using the first order and second order chiral scenarios. Comparison with staggered results. Only data T_χ (from $\chi_{\bar{\psi}\psi}$) has been used in the fits.

(unpublished !)

Conclusion for this part

Concerning the influence of the strange and charm quarks, the consensus is that the strange quark has a strong influence in the transition region,

while the charm quark does not have any influence up to about 300 MeV, where it starts contributing to the EoS.

6. The Equation of State for $N_f = 2$ and $N_f = 2 + 1 + 1$

Using the integral method. Trace anomaly for $N_f = 2$

$$
\begin{aligned}
\frac{I}{T^4} = \frac{\epsilon - 3p}{T^4} &= -\frac{T}{VT^4}\left\langle \frac{d\ln Z}{d\ln a}\right\rangle_{\rm sub} \\
&= N_\tau^4 B_\beta \frac{1}{N_\sigma^3 N_\tau}\left\{ \frac{c_0}{3}\langle {\rm ReTr}\sum_P U_P\rangle_{\rm sub} \right. \\
&\qquad + \frac{c_1}{3}\langle {\rm ReTr}\sum_R U_R\rangle_{\rm sub} \\
&\qquad + B_\kappa \langle \bar{\chi} D_{\rm W}[U]\chi\rangle_{\rm sub} \\
&\qquad \left. - \left[2(a\mu)B_\kappa + 2\kappa_c(a\mu)B_\mu\right]\langle \bar{\chi} i\gamma_5\tau^3\chi\rangle_{\rm sub} \right\}
\end{aligned}
$$

$\langle\ldots\rangle_{\rm sub} \equiv \langle\ldots\rangle_{T>0} - \langle\ldots\rangle_{T=0}$ **denotes subtraction of vacuum contributions.**

Details and results given here only for $N_f = 2$.

$N_f = 2$

F. Burger et al., Phys. Rev. D91 (2015) 074504, arXiv:1412.6748

$N_f = 2 + 1 + 1$

F. Burger et al., J. Phys. Conf. Ser. 668 (2016) no.1, 012092
arXiv:1510.02262
work still unfinished !

B_β, B_μ and B_κ are (related to) derivatives of the bare parameters with respect to the lattice spacing:

$$B_\beta = a\frac{d\beta}{da}\,, \qquad B_\mu = \frac{1}{(a\mu)}\frac{\partial(a\mu)}{\partial\beta}\,, \qquad B_\kappa = \frac{\partial\kappa_c}{\partial\beta}$$

Evaluation of the pressure by integrating the identity

$$\frac{I}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right)$$

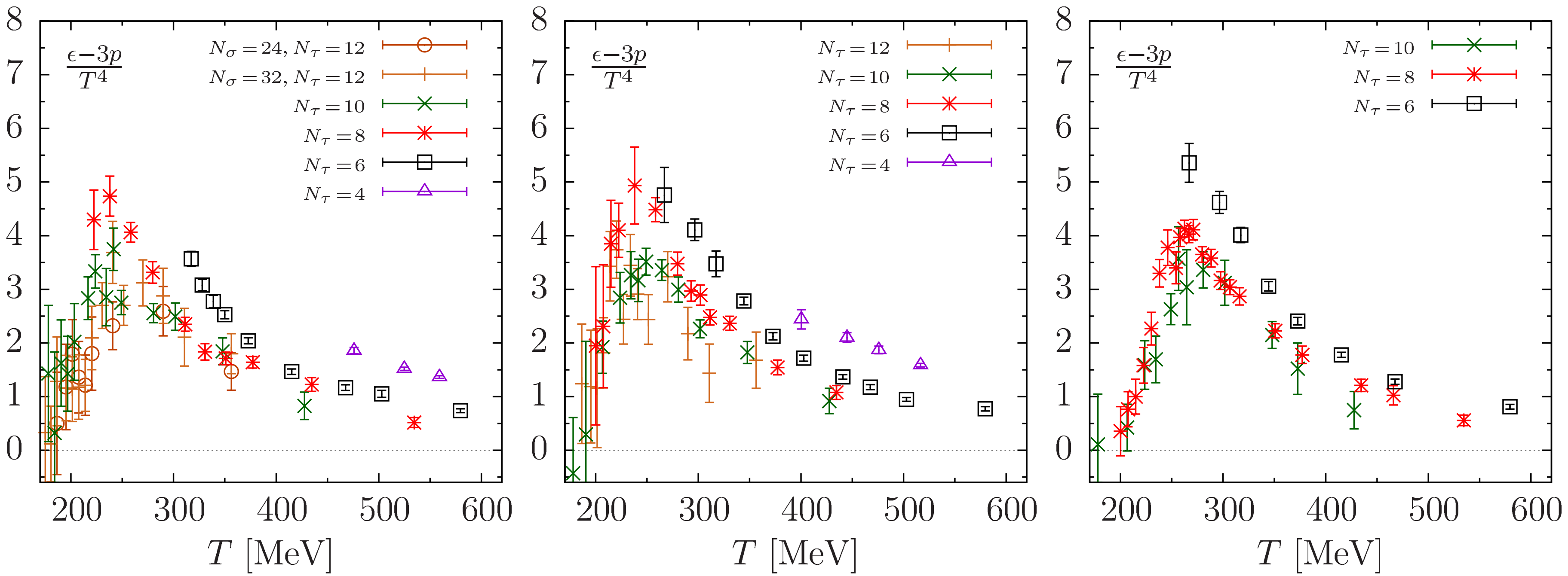
along the line of constant physics (LCP):

$$\frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} d\tau \frac{\epsilon - 3p}{\tau^5}\bigg|_{\mathrm{LCP}}$$

The available lattice data of $\frac{I}{T^4}$ have been fitted to the ansatz

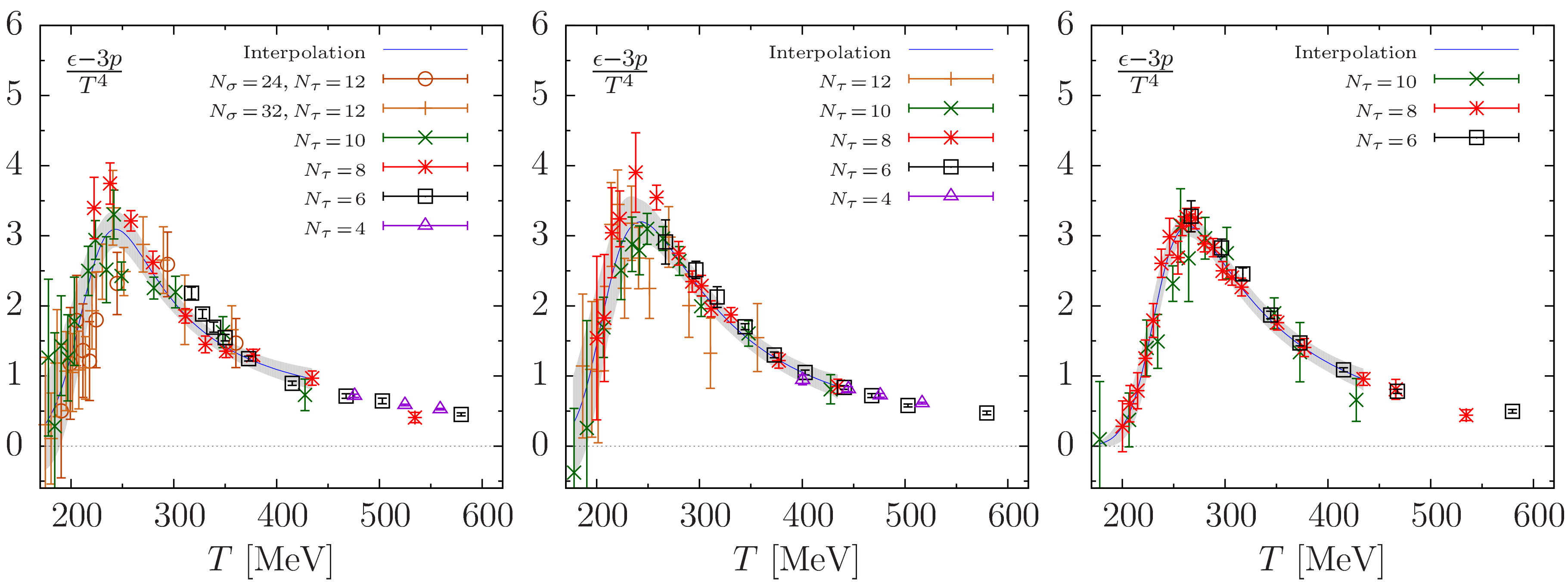
$$\frac{I}{T^4} = \exp\left(-h_1\bar{t} - h_2\bar{t}^2\right) \cdot \left(h_0 + \frac{f_0\left\{\tanh\left(f_1\bar{t} + f_2\right)\right\}}{1 + g_1\bar{t} + g_2\bar{t}^2}\right)$$

where $\bar{t} = T/T_0$ and T_0 is a further free parameter in the fit.

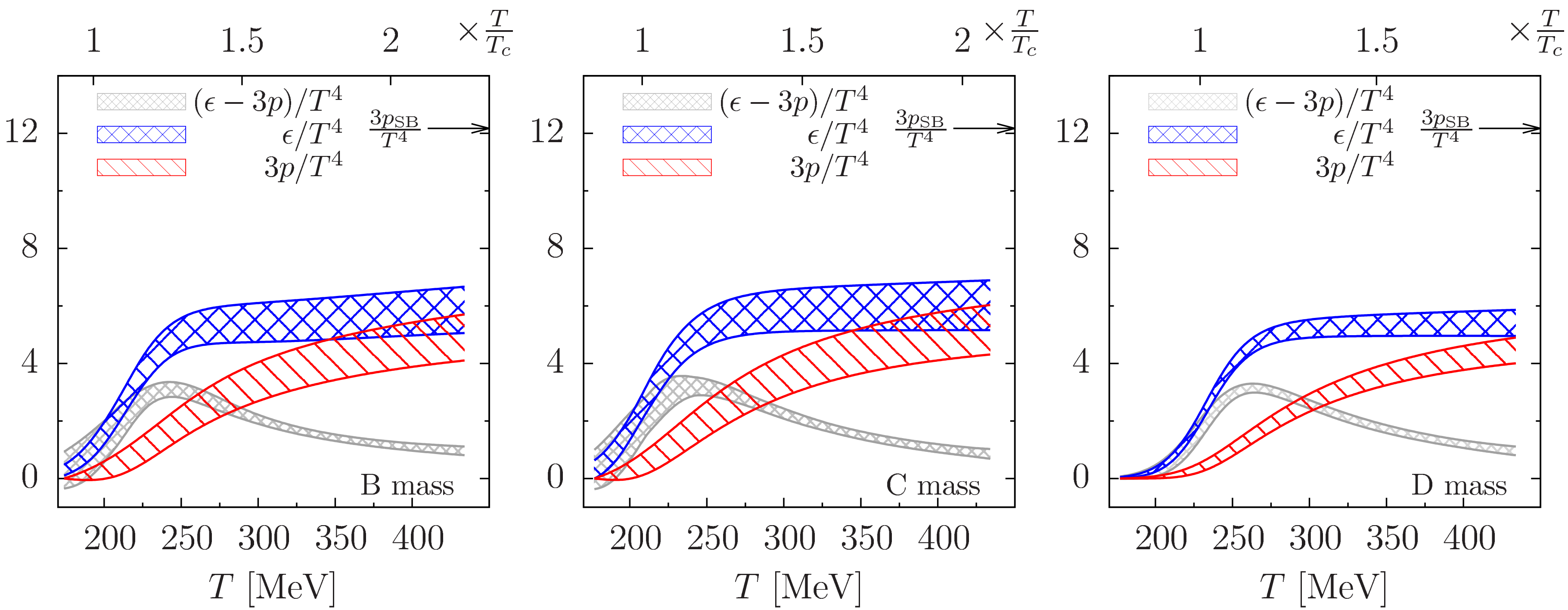


Left: Raw data for the trace anomaly for the B mass obtained for different values of the temporal extent N_τ. **Middle:** The same quantity for the C mass. **Right:** The same quantity for the D mass.

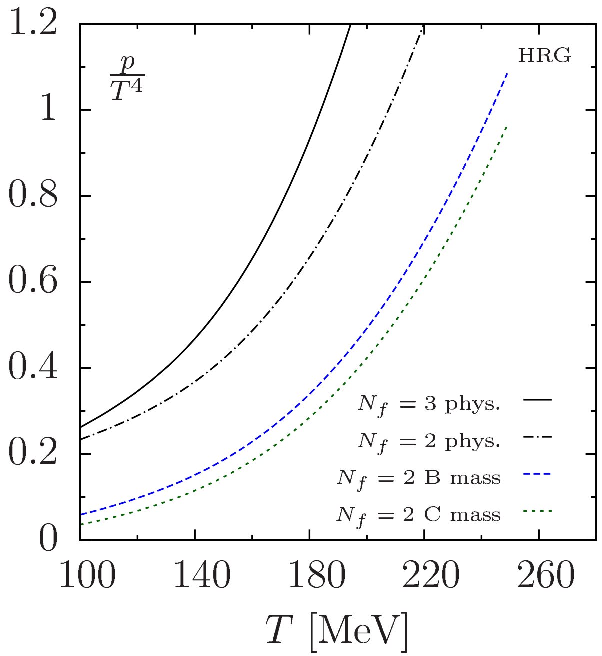
For the B mass the results obtained on the smaller spatial volume are superimposed slightly shifted for better visibility.

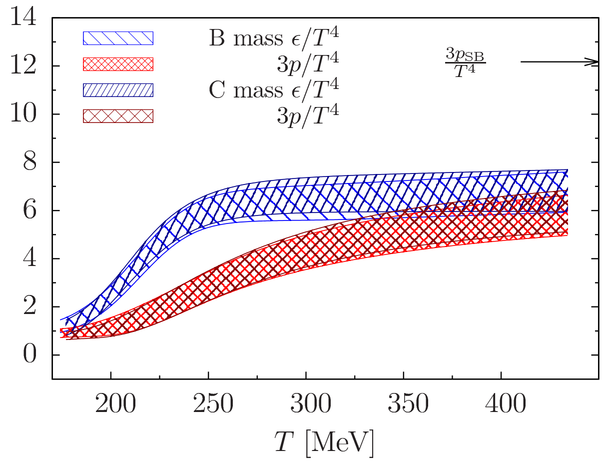


Left: The tree-level corrected trace anomaly for the B mass obtained for different values of the temporal extent N_τ ($T_\chi = 208$ MeV and $T_{\rm deconf} = 225$ MeV). Middle: The same quantity for the C mass ($T_\chi = 208$ MeV and $T_{\rm deconf} = 225$ MeV). Right: The same quantity for the D mass ($T_\chi = 229$ MeV and $T_{\rm deconf} = 244$ MeV). Also shown is a fit to all N_τ.



Left: **Final result for the pressure p and the energy density ϵ in units of T^4 for the B mass ensemble. We also show the interpolation of the trace anomaly used for integration to get the pressure. The arrow shows the expected Stefan-Boltzmann limit for the pressure.** **Middle:** **The same for the C mass.** **Right:** **The same for the D mass.**





Left: **The pressure as obtained from several HRG models is shown (different selections from the PDG table).** **Right:** **The pressure and energy density for the B mass ($m_\pi \sim 360$ MeV) and for the C mass ($m_\pi \sim 430$ MeV) as obtained when the HRG model pressure is used to fix the lower integration constant p_0.**

7. Lattice and Continuum QCD : Landau gauge Propagators at $T \neq 0$

Gauge fixing : Landau gauge

$$\nabla_\mu A_\mu(x) = \sum_\mu \left(A_\mu(x+\hat{\mu}/2) - A_\mu(x-\hat{\mu}/2)\right) = 0$$

$$A_\mu(x+\hat{\mu}/2) = \frac{1}{2iag_0}\left(U_{x\mu} - U^\dagger_{x\mu}\right)_{\text{traceless}}$$

implemented by maximization of the "gauge functional"

$$F_U[g] = \frac{1}{3}\sum_{x,\mu} \mathrm{Re}\ \mathrm{tr}\ \left(g_x U_{x\mu} g^\dagger_{x+\mu}\right)$$

with respect to suitable gauge transformations g_x.

Remarks

- Gauge dependent observables (like propagators) and gauge fixing itself didn't belong to the instruments of lattice QCD for long time, in contrast to QCD formulated in continuum (Dyson-Schwinger equations, Functional RG method).
- Both is indispensible for any productive exchange with other, non-lattice approaches to field theory and hadron physics.
- Gauge fixing leads to a multitude of copies. How to select the relevant copy (or copies) ? This is still under debate.
- If the global maximum is searched, simulated annealing helps a lot ("maximal Landau gauge").
- Finally, we went beyond the level of methodical studies !

- **Gauge fixing is performed for relevant ensembles of Monte Carlo configurations by a special algorithm, irrespective of their origin.**
- **Ghosts are not explicit, studied only algebraically, by inversion of the Faddeev-Popov operator.**
- **The quark propagator for the gluon configurations generated with twisted mass fermions by TMC at $T = 0$ has been studied in : F. Burger et al., Phys.Rev. D 87 (2013) 034514, arXiv:1210.0838**
- **We wanted to study the effect of the crossover on the gluon (and ghost) propagator, both quenched and in the presence of quarks.**
- **The quark propagator should also be studied near the crossover and compared with Dyson-Schwinger equation results. This is left to the future.**

7.1 First step: study of the quenched propagator at finite T

R. Aouane et al., Phys. Rev. D 85 (2012) 034501, arXiv:1108.1735 (with Wilson action, for various lattices)

Gluon propagator in momentum space as ensemble average :

$$D_{\mu\nu}^{ab}(q) = \left\langle \widetilde{A}_\mu^a(k) \widetilde{A}_\nu^b(-k) \right\rangle$$

$$q_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{N_\mu}\right) \qquad \text{for Matsubara frequency } q_4 = 0$$

For non-zero temperature, Euclidean invariance is broken, then it is useful to split $D_{\mu\nu}^{ab}(q)$ into two components, ($N_g = N_c^2 - 1$ and $N_c = 3$)

- **transversal D_T ("chromomagnetic") propagator**
- **longitudinal D_L ("chromoelectric") propagator**

$$D_{\mu\nu}^{ab}(q) = \delta^{ab}\left(P_{\mu\nu}^T D_T(q_4^2, \vec{q}^{\,2}) + P_{\mu\nu}^L D_L(q_4^2, \vec{q}^{\,2})\right)$$

Propagators $D_{T,L}$ (or their respective dimensionless dressing functions $Z_{T,L}(q) = q^2 D_{T,L}(q)$) obtained from the Fourier transforms

$$D_T(q) = \frac{1}{2N_g} \left\langle \sum_{i=1}^{3} \widetilde{A}_i^a(k)\widetilde{A}_i^a(-k) - \frac{q_4^2}{\vec{q}^{\,2}} \widetilde{A}_4^a(k)\widetilde{A}_4^a(-k) \right\rangle$$

and

$$D_L(q) = \frac{1}{N_g} \left(1 + \frac{q_4^2}{\vec{q}^{\,2}}\right) \left\langle \widetilde{A}_4^a(k)\widetilde{A}_4^a(-k) \right\rangle$$

The corresponding renormalized functions, in momentum subtraction (MOM) schemes, can be obtained from

$$Z_{T,L}^{ren}(q,\mu) \;\equiv\; \tilde{Z}_{T,L}(\mu) Z_{T,L}(q),$$

with the $\tilde{Z}$-factors being defined such that $Z_{T,L}^{ren}(\mu,\mu) = 1$.

Fitting gluon dressing functions we used the Gribov-Stingl form

$$Z_{\rm fit}(q) = q^2 \frac{c\;(1 + d\, q^{2n})}{(q^2 + r^2)^2 + b^2}$$

Our main emphasis : Finite-volume and discretization studies,

providing continuum parametrizations for various temperatures, to be used as input (or benchmark) for finite-T continuum studies (performed by DS equations or Functional RG studies)

- **Gribov copy and finite volume effects turned out to be of minor importance in the momentum range under study**
- **fits of the momentum dependence of the propagators**
  $0.6\ GeV < q < 3.0\ GeV$
- **in the temperature range**
  $0.65 < T/T_{\mathrm{deconf}} < 2.97$

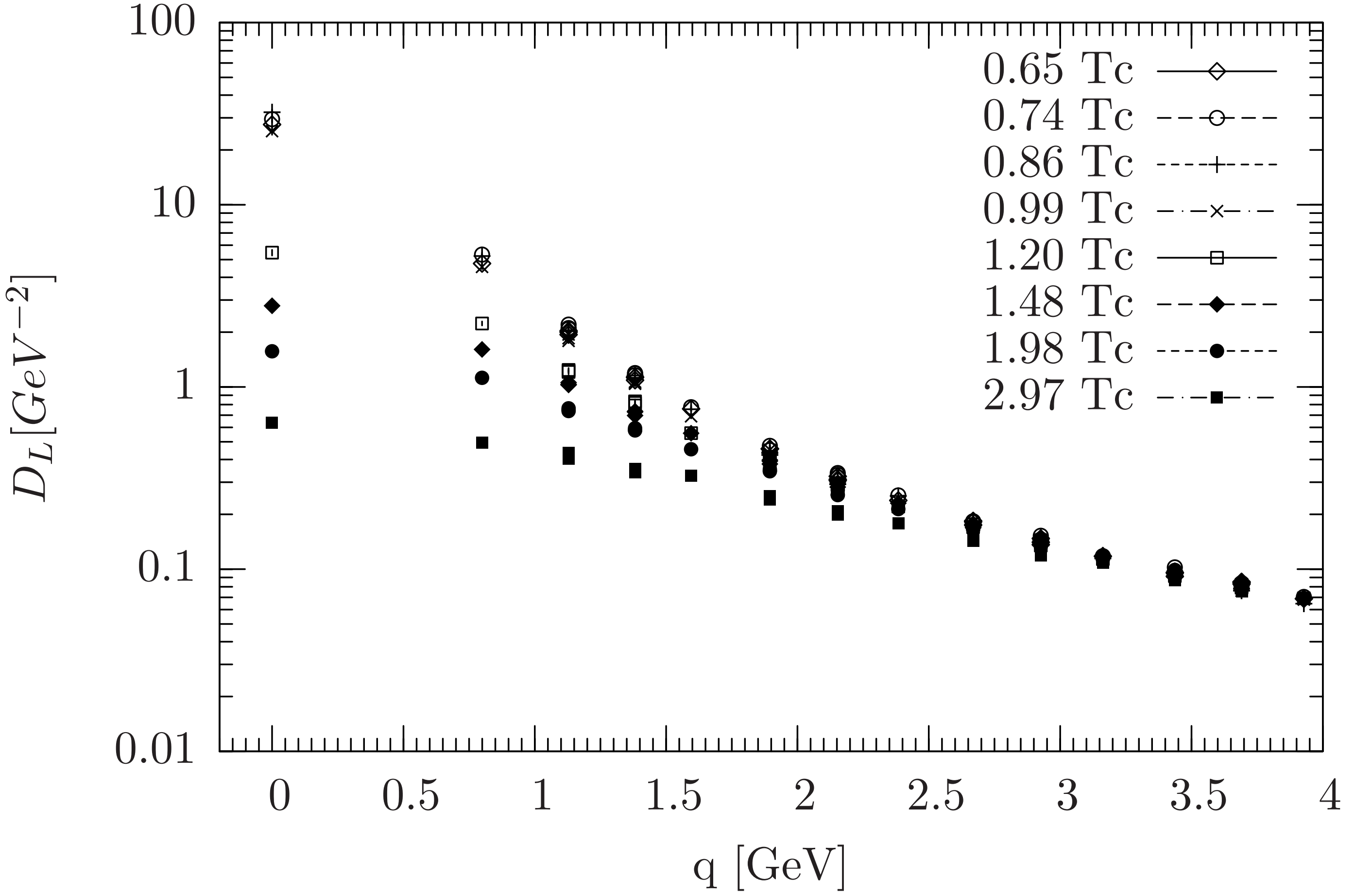
Important : the first order nature of the phase transition is evident. More or less rapid changes are visible in the propagators !

Temperature dependence studied at fixed scale, here shown only for $\beta = 6.337$, the finest lattice

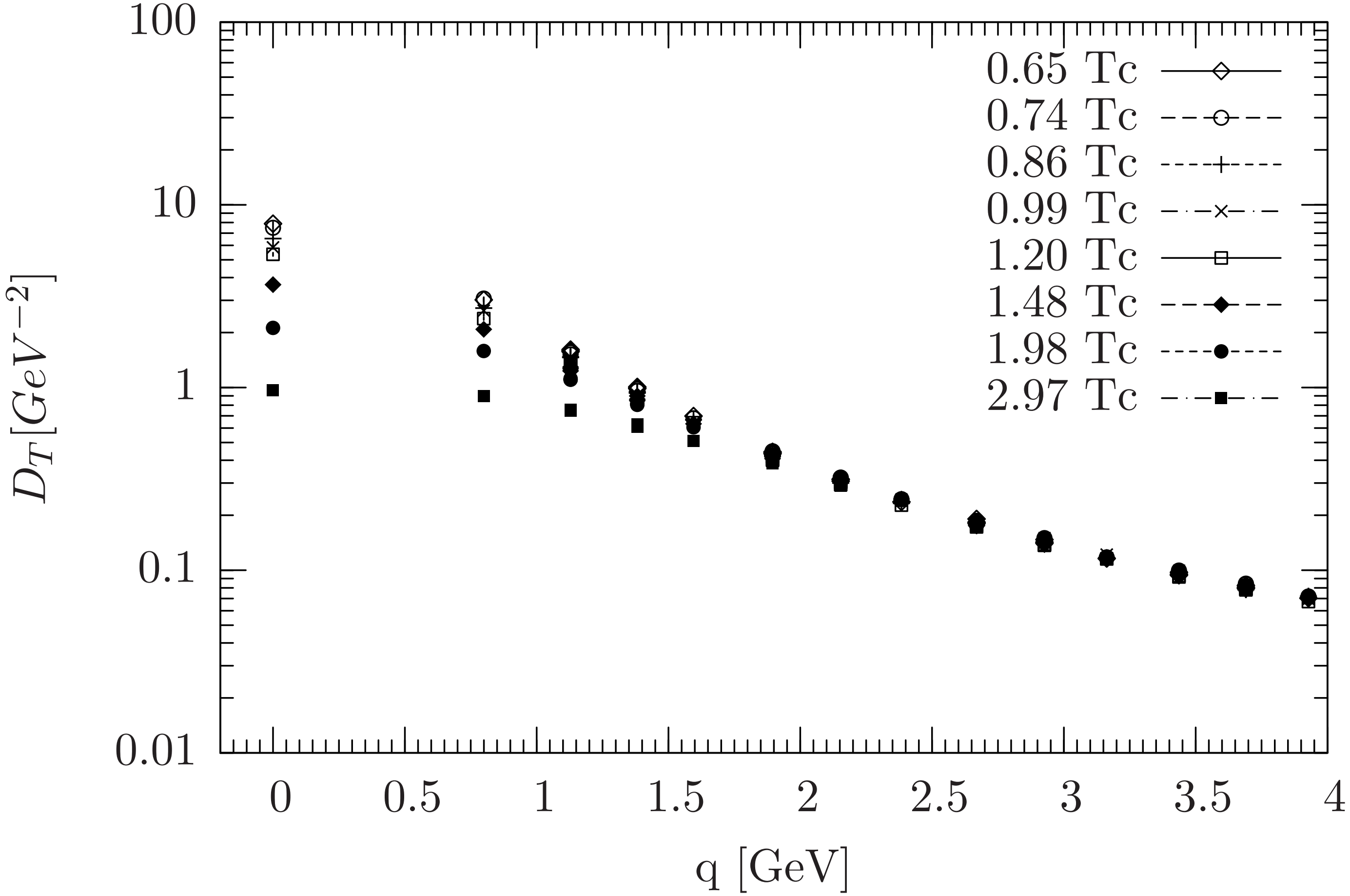
T/T_c	N_τ	N_σ	β	$a(\mathrm{GeV}^{-1})$	$a(\mathrm{fm})$	n_{conf}	n_{copy}
0.65	**18**	**48**	**6.337**	**0.28**	**0.055**	**150**	**1**
0.74	**16**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
0.86	**14**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
0.99	**12**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.20	**10**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.48	**8**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
1.98	**6**	**48**	**6.337**	**0.28**	**0.055**	**200**	**1**
2.97	**4**	**48**	**6.337**	**0.28**	**0.055**	**210**	**1**

long simulated annealing sequences, no copies !

q dependence of color–electric D_L for various temperatures



q dependence of color–magnetic D_T for various temperatures



7.2 Phase structure from propagators ?

Is it difficult to reconcile the first order phase transition with the (only) gradual changes of the propagators actually seen ? No !

Our finite-temperature results for pure Yang-Mills theory have been used by K. Fukushima and K. Kashiwa, Phys. Lett. B 723 (2013) 360 arXiv:1206.0685v5

- **for the effective potential of the Polyakov loop**
- **for reconstructing the Equation of State (EoS)**

In leading order of the 2PI formalism, the thermodynamical potential can be approximated as follows in terms of both the gluon and ghost propagators :

$$\frac{1}{T}\Omega_{\mathbf{glue}} \simeq -\frac{1}{2}\mathrm{tr}\ \ln D_{\mathrm{gl}}^{-1} + \mathrm{tr}\ \ln D_{\mathrm{gh}}^{-1}$$

For example, the inverse gluon propagator have been extracted from our data in the interval up to $T = 1.2\ T_c$

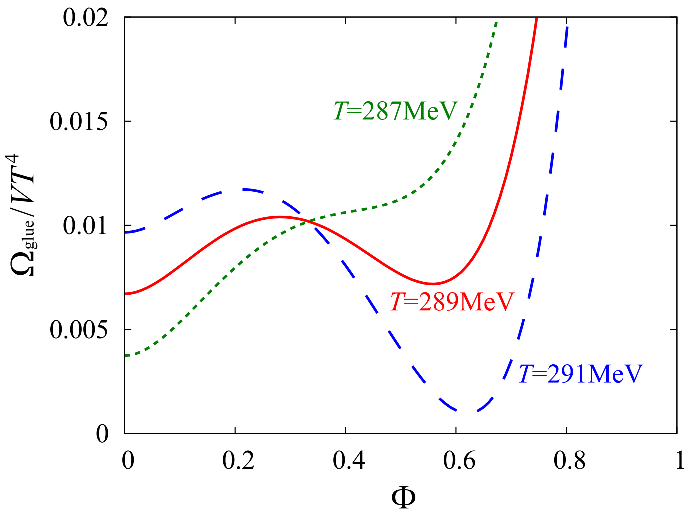
$$D_{\rm gl}^{-1}(p^2) = \left[p^2 Z_T(p^2) T_{\mu\nu} + \xi^{-1} p^2 Z_L(p^2) L_{\mu\nu} \right] \delta^{ab}$$

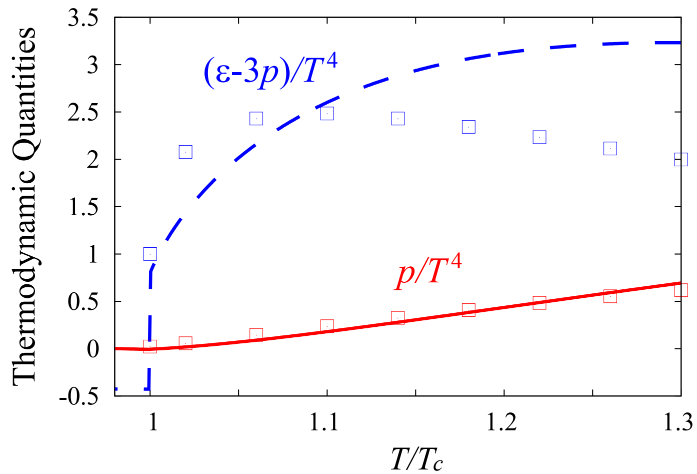
Results :

- **Order and transition temperature has been successfully reconstructed from our T-dependent propagator data.**
- **The pressure and trace anomaly are only qualitatively obtained.**

Order parameter and EoS of pure Yang-Mills (Fukushima and Kashiwa)

Transition temperature and first rise of the pressure successfully reconstructed from our T-dependent propagator data for gluodynamics !





7.3 DSE prediction for unquenching the propagators

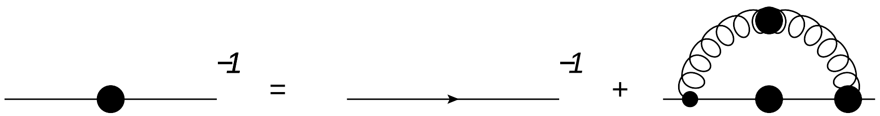
Can one obtain non-quenched propagators from the quenched ones without actually doing the non-quenched lattice simulation ?

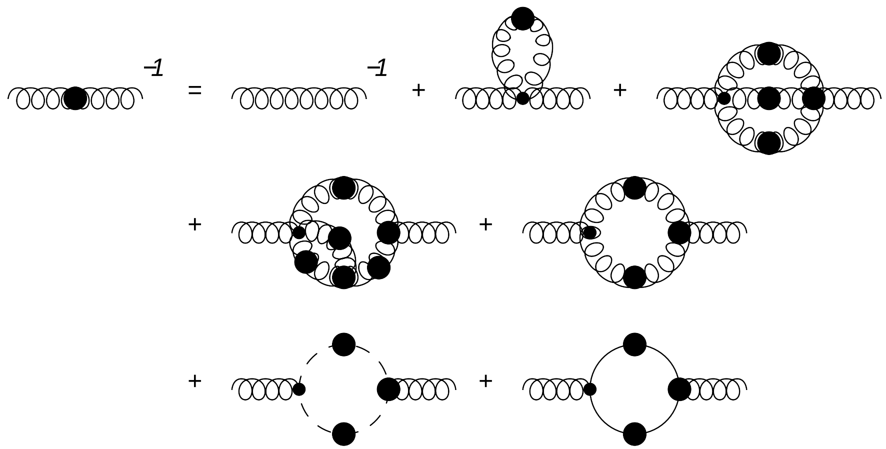
How good can DSE predict/postdict what will be/has been measured on the lattice in a non-quenched simulation ?

C. S. Fischer and J. Luecker, "Progagators and phase structure of $N_f = 2$ and $N_f = 2 + 1$ QCD"
Physics Letters B 718 (2013) 1036, arXiv:1206.5191,
and arXiv:1306.6022

The full set of Dyson-Schwinger equations was used to predict the T dependence of full QCD propagators from the quenched ones, depending on m_π as a parameter to characterize the non-quenched simulations.

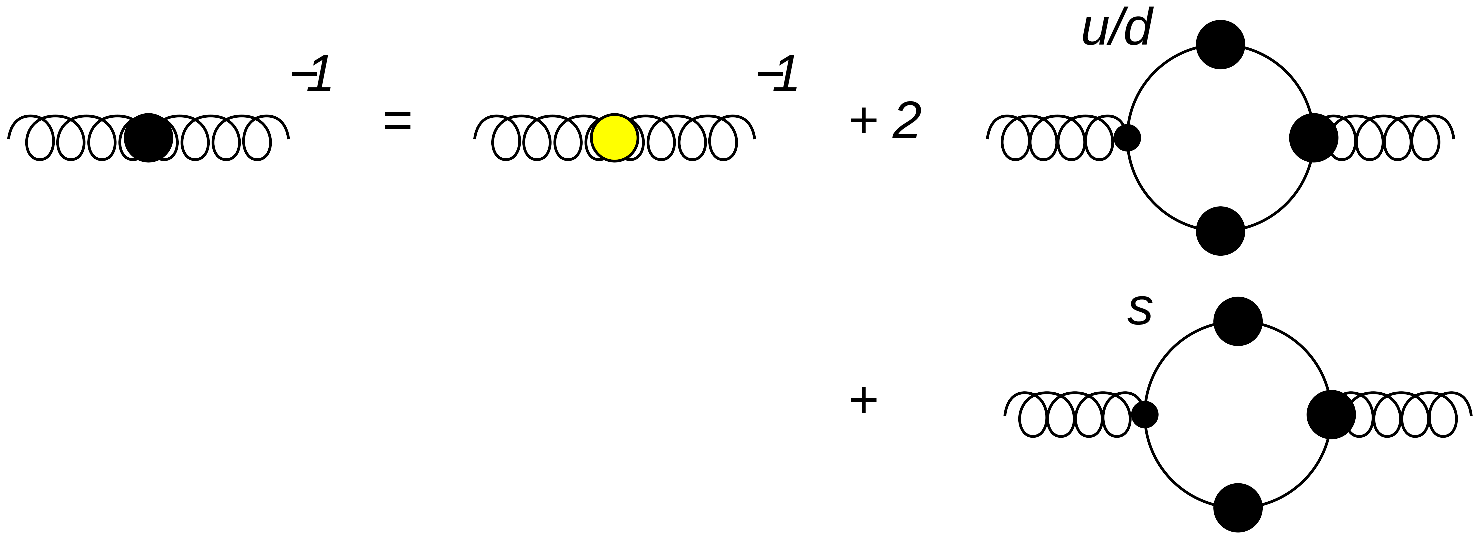
Full Dyson-Schwinger equations for the quark (left) and the gluon (right) propagator





Truncated gluon Dyson-Schwinger equation relating the quenched and the non-quenched gluon propagator (for u, d and evtl. s quarks)

(yellow insert = quenched gluon propagator)



By-product of this study : quark propagator at $T \neq 0$

(was not yet provided by us for twisted mass at $T \neq 0$)

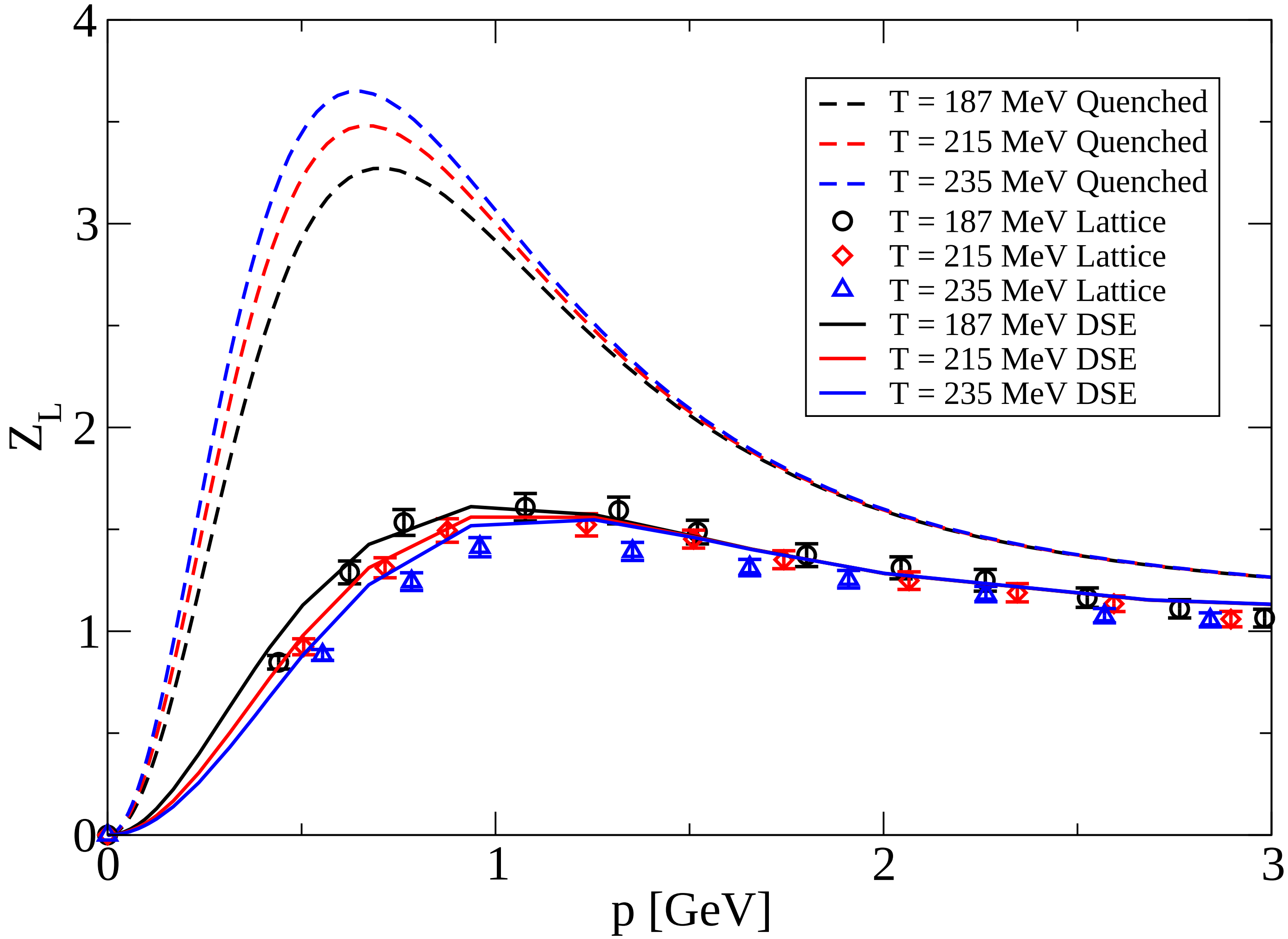
The quark propagator is planned to be measured in future finite-T simulations (now with $N_f = 2 + 1 + 1$).

Will be interesting to compare with DSE predictions !

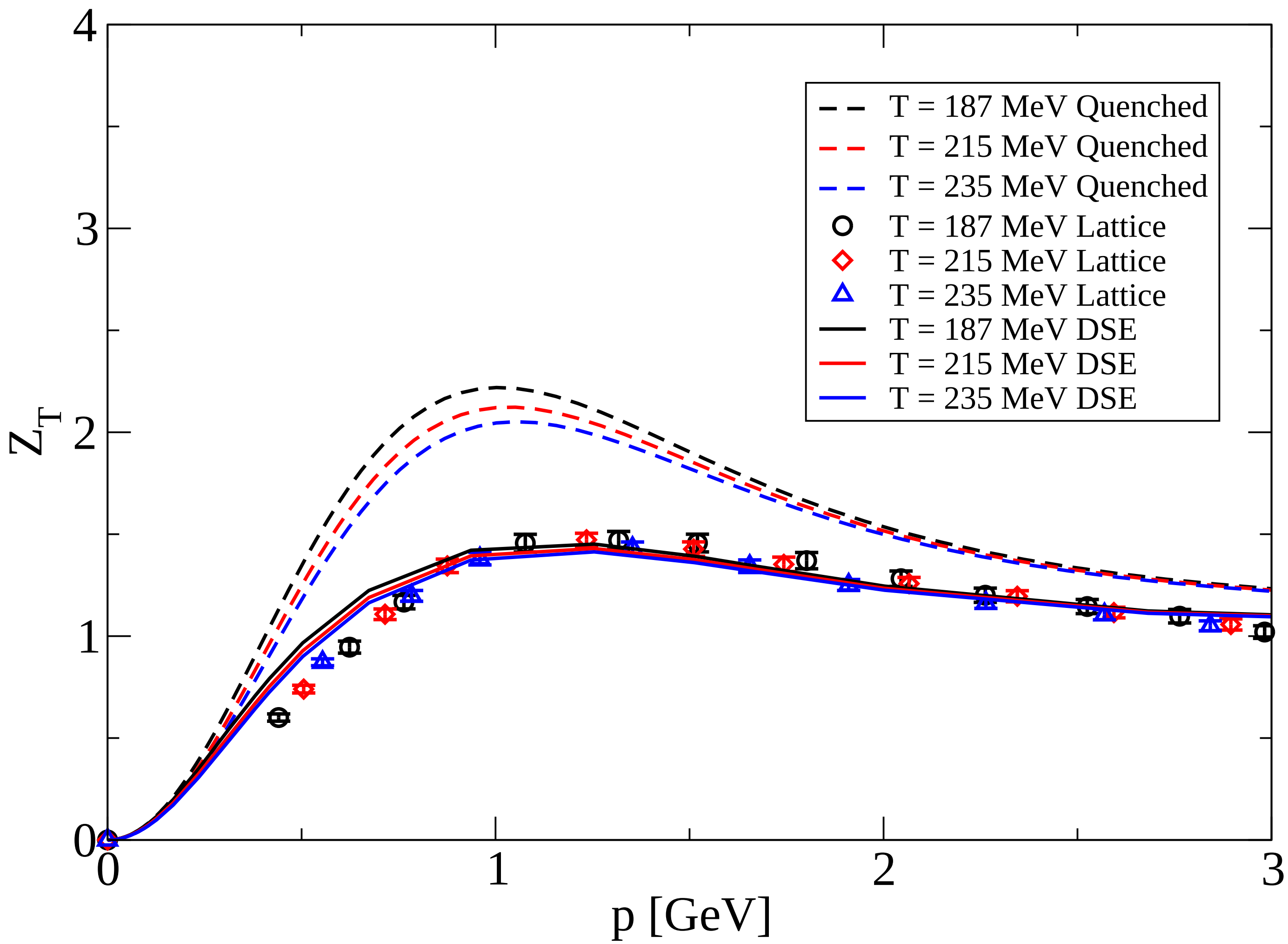
Ch. Fischer (Giessen) strongly interested in availability of $N_f = 2 + 1 + 1$ data !

An *ab initio* study of the (momentum dependence) of the quark-gluon vertex is presently of high interest !

q dependence and unquenching effect of Z_L for various temperatures

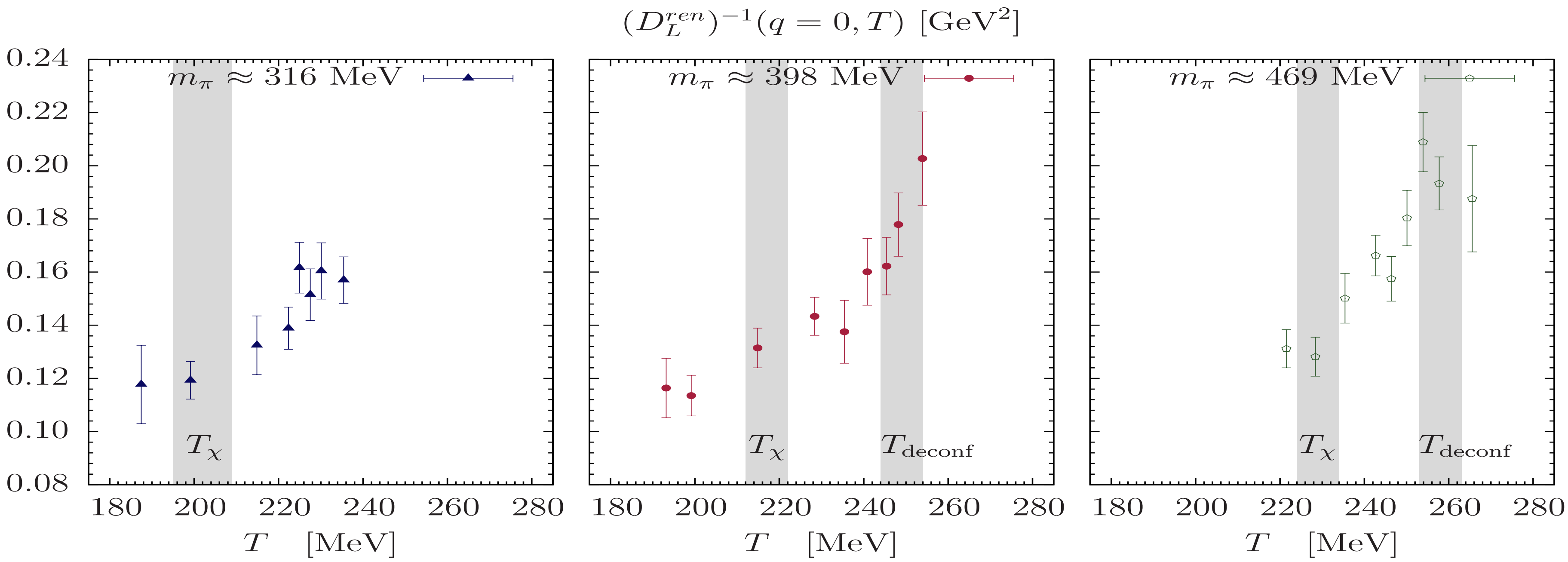


q dependence and unquenching effect of Z_T for various temperatures

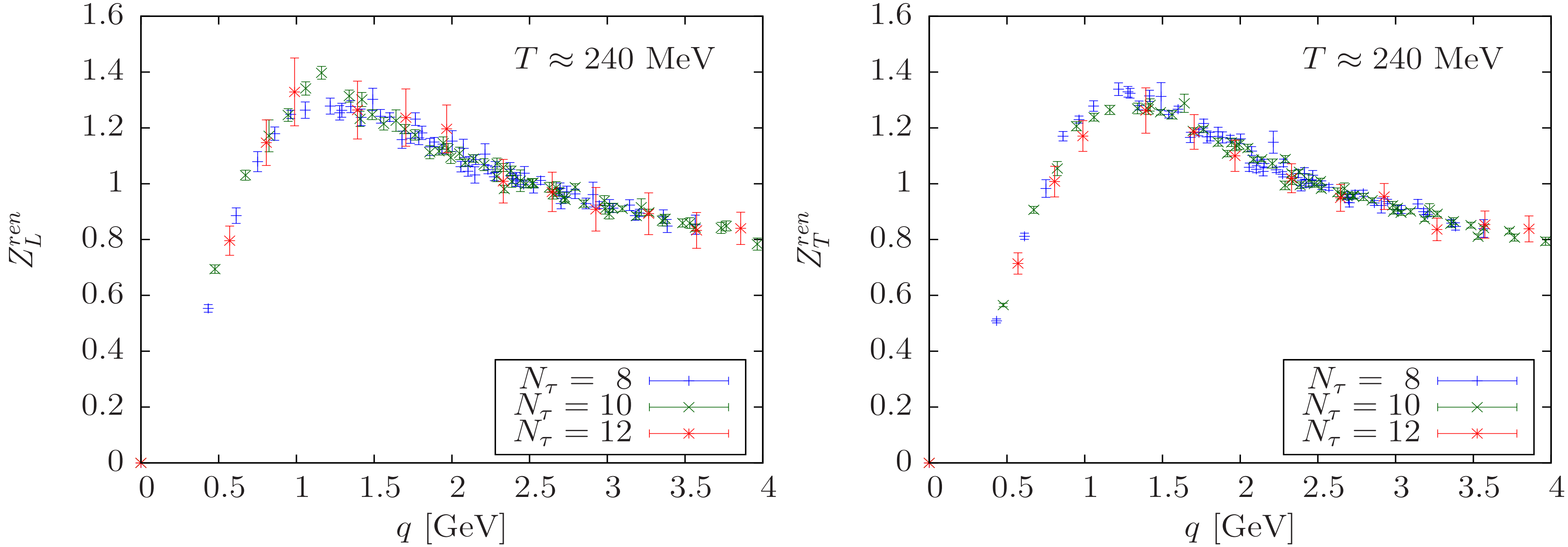


Our next paper Phys. Rev. D 87 (2013) 114502, arXiv:1212.1102 "Landau gauge gluon and ghost propagators from lattice QCD with $N_f = 2$ twisted mass fermions at finite temperature"
R. Aouane, F. Burger, E.-M. I., M. Müller-Preussker, A. Sternbeck
has provided the awaited unquenched propagators for twisted mass ensembles of the tmfT collaboration, in continuum parametrizations ready for comparison with DSE predictions in the momentum ranges :

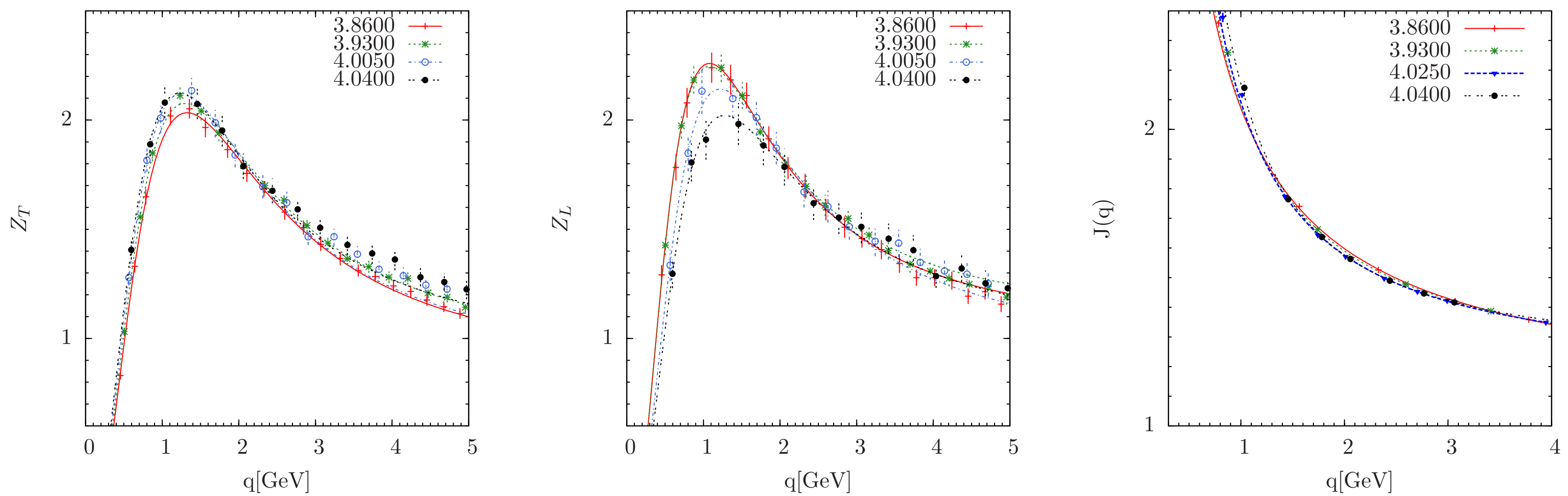
- $0.4\ GeV < q < 3.0\ GeV$ **for the gluon propagators (perfect fit !) fitting parameter b^2 in the Grivov-Stingl fit compatible with zero (no splitting in complex conjugate poles visible in this momentum range !)**
- $0.4\ GeV < q < 4.0\ GeV$ **for the ghost propagator (less good, fit correct within few percent, a mass term m_{gh} wouldn't help),**



The panels show the inverse renormalized longitudinal gluon propagator $(D_L^{ren})^{-1}$ at zero momentum for the three pion mass values indicated.



The left (right) panel shows a scaling check of the renormalized longitudinal (transversal) gluon dressing function at $T \simeq 240$ MeV by varying $N_\tau = 8, 10, 12$. (For $m_\pi \simeq 398$ MeV as an example !)



The unrenormalized dressing functions for the transverse gluon Z_T (left), for the longitudinal gluon Z_L (middle) and for the ghost dressing function J (right) are shown as functions of the momentum q $[\mathrm{GeV}]$ for different values of β (i.e. different temperatures) given in the legend. (The corresponding pion mass value is $m_\pi \simeq 398$ MeV as an example.)

8. Perspectives of the Project: new Questions/new Partners for Dubna

1. **Frascati: M. P. Lombardo, not a new partner !**
   with contributions from A. Trunin (Dubna) and F. Burger (Berlin)
   - **closer characterization of the thermal crossover for** $N_f = 2+1+1$
   - **topological susceptibility** $\chi_{\rm top}(T)$ **for** $N_f = 2+1+1$
   - **equation of state with the integral method for** $N_f = 2+1+1$

   Emphasis on topology and EoS !

2. Heidelberg: J. M. Pawlowski and A. Rothkopf with input (gauge fixing) from A. Trunin (Dubna)

- **usual Landau gauge gluon propagator for $N_f = 2 + 1 + 1$ (zero Matsubara frequency, already evaluated for the ensembles with largest $m_{\pi^{\pm}}$.)**
- **spectral functions from Landau gauge gluon correlation functions, using a new Bayesian technique [Rothkopf]**
- **complex heavy-quark potential from Wilson-line correlators in Coulomb gauge [Rothkopf]**

Emphasis on real-time properties of QGP !

3. Giessen and Jena : Ch. Fischer and A. Sternbeck

- **exploring the phase structure through propagators and vertex functions**
- **effect of changing N_f via SDE**
- **extension to $\mu \neq 0$ through SDE**
- **thus finding the critical endpoint**
- **begin a systematic study of the quark propagator**
- **begin a systematic study of the gluon-quark vertex**

Emphasis on phase structure for different N_f, for $\mu \neq 0$, quark propagator and spectral function !

4. Giessen : L. von Smekal

- **further studies of thermodynamic functions**
- **Taylor expansion for small** $\mu \neq 0$
- **calculation of transport coefficients in QGP**

Empasis on properties inside the QGP !

9. Conclusions and Outlook

- The $N_f = 2$ crossover structure and its chiral limit are understood.
- The results are in fair agreement with other results with Wilson fermions and staggered fermions.
- As expected for a crossover, chiral and deconfinement crossover temperatures are not exactly coincident.
- The effect of unquenching on the gluon propagators (longitudinal and transversal) had been successfully predicted by DSE (Ch. Fischer et al.) on the basis of our previous quenched measurements.

- **The effect of the first order transition in gluodynamics ($N_f = 0$) and of the crossover with $N_f = 2$, respectively, on the gluon propagators is manifest mainly in the longitudinal (electric) gluon propagator.**
- **A rapid drop of D_L at low momenta in the high-temperature phase starts not before $T > T_\chi$ (spans two orders of magnitude).**
- **A softer drop of D_T at low momenta begins already below T_χ (only one order of magnitude is spanned).**
- **The effect on the effective potential of the Polyakov loop (in the case of gluodynamics) results from the T dependence of both D_L and D_T**
- **An approx. calculation of the EoS has been tried, needs to be refined.**
- **Interesting is the effect of heavier quarks with the extension from $N_f = 2 \to N_f = 2+1+1$ (both available m_π from 470 MeV down to 210 MeV).**

- **Interesting cross-checks with Dyson-Schwinger (Ch. Fischer et al. concerning N_f dependence, extension to $\mu \neq 0$) and with other continuum approaches are possible through the propagators !**
- **Strange and charmed quarks are less important for finding the chiral transition temperature in the chiral limit ($m_u = m_d \to 0$).**
- **However, strange quarks influence the actual temperature of the crossover for (near to) physical light quark masses.**
- **Strange and charmed quarks are essential for the thermodynamics (EoS and other thermodynamic functions) of the high-temperature phase !**
- **Gluon and quark propagators should be studied in order to check the N_f dependence predicted by the Dyson-Schwinger approach. The prediction from $N_f = 0$ to $N_f = 2$ (unquenching) was successful.**

- $N_f = 2+1+1$ **configurations are under examination (together with quenched lattice configurations) in order to study the non-positive gluon spectral function in both cases (with J. Pawlowski and A. Rothkopf).**

- $N_f = 2+1+1$ **configurations are under examination to what extent the complex-valued** $\bar{Q}Q$ **potential can be extracted (with A. Rothkopf).**

- **The high-temperature dependence of the topological susceptibility** $\chi_{\rm top}(T)$ **is under study.**

- **Spacelike hadron propagators (screening lengths in various channels) are recommended to characterize the restoration of symmetries.**

- **Hadron spectral functions: Euclidean** $\rightarrow$ **Minkowski time correlators.**